When is Nonfundamentalness in SVARs a Real Problem?

Paul Beaudry
Vancouver School of Economics, University of British Columbia and NBER

Patrick Fève*
Toulouse School of Economics and University of Toulouse I-Capitole

Alain Guay
Université du Québec à Montréal and CIREQ

Franck Portier
University College London and CEPR

February 2019

Abstract

In SVARs, identification of structural shocks can be subject to nonfundamentalness, as the econometrician may have an information set smaller than the economic agents' one. How serious is that problem from a quantitative point of view? In this paper we propose a simple diagnostic for the quantitative importance of nonfundamentalness in structural VARs. The diagnostic is of interest as nonfundamentalness is not an either/or question, and its quantitative implications can be more or less severe. As an illustration, we apply our diagnostic to the identification of TFP news shocks and we find that nonfundamentalness is of little quantitatively importance in that context.

Key Words : NonFundamentalness, Business Cycles, SVARs, News Shocks.

JEL Class. : C32, E32

---

*Corresponding author: Patrick Fève, TSE-Université Toulouse I-Capitole, Manufacture des Tabacs, bat. F, 21 allée de Brienne, 31000 Toulouse, France (e-mail: patrick.feve@tse-fr.eu). We thank the Editor, the Associate Editor and two anonymous referees from their remarks. We also thank Fabrice Collard, Filippo Ferroni, Mario Forni, Luca Gambetti, Morten Ravn and Luca Sala for valuable suggestions. Forni, Gambetti and Sala kindly shared their codes and data with us. This paper has benefited from helpful discussions during presentations at various seminars and conferences. This project has received funding under the European Union's Horizon 2020 research and innovation programme (ADEMU grant agreement No 649396).

Introduction

Since Sims [1980], Structural Vector AutoRegressions (SVARs) have become a popular tool for macroeconomists, as they allow to identify the structural shocks that affect the macroeconomy as well as the response to those shocks (see Ramey [2016] and Kilian and Lütkepohl [2017] for a complete review). In a comment on Blanchard and Quah's [1989] SVAR exercise, Lippi and Reichlin [1993] raised the question of the nonfundamentalness of some structural moving average representations. When the econometrician has less information than the agents in the economy, she might not recover the structural shocks from the present and past observations of the economy regardless the identification strategy. In such a case, the moving average representation is nonfundamental. The example given by Lippi and Reichlin [1993] and further developed by Lippi and Reichlin [1994] is the one of a technological diffusion process, for which economic agents act knowing the future development of technology while the econometrician does not have such an information.

If one believes that Dynamic Stochastic General Equilibrium (DSGE) models are a good approximation of the true data generating process, then nonfundamentalness might be more than a theoretical curiosity. Indeed, Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007] have shown that DSGE models may not have a fundamental moving average representation in the structural shocks, so that a SVAR cannot recover the structural shocks. From a quantitative perspective, Sims [2012] then shown that nonfundamentalness is not so much of a either/or problem: there are models in which one can pretty well, if not perfectly, recover structural shocks even with nonfundamentalness, as the information of the econometrician "almost" includes the one of the economic agents.

The questions then becomes an empirical one: can we test whether or not a structural representation of the data is fundamental? Forni and Gambetti [2014] and Forni, Gambetti, and Sala [2014] have suggested to answer this question by testing for the orthogonality of SVAR residuals to a large information set that is well captured by the main factors of a Factor Augmented VAR (FAVAR) model – *i.e.* a VAR model to which is added the main factors of a large model with hundreds of macroeconomic variables, that is likely to contain all the information possessed by economic agents. The "sufficient information" test can detect wether or not the SVAR suffers from nonfundamentalness (under the assumption that the factors contain all the information that is used by the economic agents).[1]

[1] Canova and Hamidi Sahneh [2016] have investigated the robustness of the Forni and Gambetti [2014] approach. Tests of fundamentalness have also been proposed by Chen, Choi, and Escanciano [2012] and Hamidi Sahneh [2015]

But as for theory, an either/or test for nonfundamentalness is of limited interest, as it does not tell whether the consequences of nonfundamentalness are severe or not. The paper first proposes an empirical diagnostic of the nonfundamentalness severity. We show that the coefficient of determination (hereafter R^2) of the projection of innovations of SVARs on factors is indeed a proper measure on that severity. The problem is to judge how large the R^2 has to be for the consequence of nonfundamentalness to be severe. Simulation experiments conducted in Sections 2.2 and 2.3 show that autoregressions yields accurate dynamic responses for a R^2 below 0.25. Interestingly, this R^2 has some tight connections with some previous literature on VARs and identification, as we will show that it is a measure of the "anticipation rate" discussed in linear rational expectations models by Ljungqvist and Sargent [2004] and Mertens and Ravn [2010]. It is also directly related to the "Poor Man's Invertibility Condition" of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007], more specifically to the largest eigenvalue of $A - BD^{-1}C$ matrix (using the "ABCD" language of these scholars). An additional contribution of the paper is to explicitly characterize the bias in estimated dynamic responses obtained from a misspecified SVARs (in the sense that it omits the relevant state variables, represented as a set of factors) in terms of R^2. We analytically determine the link relation between the R^2 and the upper bounds of the relative bias in estimated impact responses obtained from a bivariate SVAR setup with the very commonly used Cholesky decomposition.

We then implement our R^2 diagnostic in the case of the identification of technological news shocks. The connection between news shocks and nonfundamentalness is tight: if agents receive some information about future technological improvements, this information might not be embedded in the current information set of the econometrician. The running example of Lippi and Reichlin [1994] when they illustrated nonfundamentalness was indeed a technological diffusion. A key insight of the "news" VARs of Beaudry and Portier [2006] is that the use of asset prices might overcome the nonfundamentalness problem, as they are likely to react strongly to agents' changing views of the future. As Forni, Gambetti, and Sala [2014] have questioned this property and shown that such identified technological news might be tested as nonfundamental, it is of interest to implement our R^2 diagnostic in this case. As we will show, relevant R^2 range between 3% and 21% depending on the specification, and the consequences of nonfundamentalness appear to be of relative minor importance in practice.

Two modeling issues deserve additional comments. First, we consider that factors (estimated by the econometrician) span the true state of the economy, as usual in that literature (see Stock and

for non-Gaussian structural shocks. See Kilian and Lütkepohl [2017] for a discussion of the non-Gaussian case.

Watson [2016]). Under this assumption, we are armed with a simple diagnostic about potential misspecification of the SVAR model. As shown in Forni, Giannone, Lippi, and Reichlin [2009], this augmented setup is less (if not) affected by the nonfundamentalness problem, because it includes a sufficient amount of information. Second, our procedure assumes that the omitted factors are known and one may wonder why they are not directly included in the VAR model. Our approach has the advantage of maintaining a parsimonious (small scale) VAR model and thus does not require estimating a large number of parameters. A small scale VAR model allows minimizing the root mean square errors of the estimated impulse responses. This is why it is preferred by applied researchers. However, it can suffer from an omitted variables problem and thus yield inconsistent estimates of shocks (see Canova [2007]). With our pre-test procedure, we avoid this problem because our diagnostic allows to check if a small-scale SVAR model is a proper approximation of the true dynamic structure. In addition, the R^2 diagnostic could be employed as an information criterion to properly select a limited set of relevant variables in the VAR model and thus to recover the structural shocks of interest.

As related literature, Soccorsi [2016] develops a global measure of nonfundamentalness with respect to DSGE models. The measure is a distance based in covariances between the true nonfundamental innovations and the innovations resulting from the unique fundamental representation obtained by flipping the problematic roots of the MA (Moving Average) structural representation. Forni, Gambetti, and Sala [2016] also develop a measure of nonfundamentalness for a specific structural shock by projecting this shock onto the VAR innovation. Both measures are implemented to evaluate the severity of the nonfundamentalness problem respective to specific theoretical macroeconomic models and have the property to disentangle the nonfundamentalness bias from the lag truncation bias resulting from a finite VAR. However, the application of these measures of nonfundamentalness necessitates the knowledge of the DSGE model. We develop a similar population measure of the severity of the nonfundamentalness for the whole system or for a single structural shock for macroeconomic models that can be expressed in state-space representation. The original contribution of the paper is to show that our R^2 diagnostic can be directly implemented on observable data using SVARs without specifying any DSGE models, under the assumption that the factors well capture the state variables of the economy. The R^2 diagnostic can then detect empirically the severity of nonfundamentalness and/or lag truncation problems of the finite VAR model.

The paper is organized as follows. In a first section, we expound the $ABCD/AKC\Sigma$ setups and the R^2 diagnostic. We also illustrate the merits of this diagnostic using a simple Lucas' tree model. In a second section, we perform quantitative and simulation experiments. A third section connects the bias that arises from a misspecified VAR model to the R^2. In the fourth section, we implement the R^2 diagnostic in the case of the identification of technological news shocks with US data. A last section concludes. Proofs are reported in appendix.

1 $ABCD/AKC\Sigma$ Setups and the R^2 Diagnostic

In this section, we introduce notations for a structural model, its VAR representation and the conditions under which that model is invertible, so that its structural moving average representation is fundamental.[2] We will use the "$ABCD$" language of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007]. Then we will introduce a simple economic example and show how a properly defined R^2 statistics can be informative of the nonfundamentalness severity.

1.1 $ABCD/AKC\Sigma$ Setups

Let us consider the following state-space representation

$$x_t = Ax_{t-1} + B\varepsilon_t \tag{1}$$

$$y_t = Cx_{t-1} + D\varepsilon_t, \tag{2}$$

where x_t is a vector of state variables, y_t a vector of observed variables and ε_t a vector of a white noise structural shocks distributed as a normal distribution with normalized variance. Here we assume that D is invertible.[3] The question is then whether or not one can retrieve the true dynamic and stochastic structure of (1)–(2) from the observation of y_t only. We use here an important result from Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007].

Proposition 1 *A sufficient condition for invertibility is that all the eigenvalues of* $(A - BD^{-1}C)$ *are less than one in modulus.*

This sufficient condition that all the eigenvalues of $(A - BD^{-1}C)$ are less than one in modulus is the "poor man invertibility" condition given in Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson [2007]. If Proposition 1 is satisfied, the model (1)–(2) has a VAR representation in y_t

[2] Fundamentalness is closely related to the concept of invertibility. Invertibility requires that no root of the determinant of the moving-average representation is on or inside the unit circle. Fundalmentalness requires that no root is inside the unit circle.

[3] There exist different situations for which the matrix D is noninvertible. For example, if we assume that shocks are imperfectly observed by private agents, i.e. they receive a noisy signal on the fundamentals, they can not disentangle the true shock from the noise and the matrix D is singular.

and it is possible to uncover the structural shocks.[4] However, using optimal forecasts of the state variables from the vector of observed variables, we can construct a VAR representation of y_t. We denote this representation as the $AKC\Sigma$ innovation representation. K and Σ represent the Kalman gain and the variance of the forecast error $\Sigma = E\left((x_t - \hat{x}_t)(x_t - \hat{x}_t)'\right)$, i.e. the optimal forecast of the state vector x_t given the observations up to y_t. The matrices K and Σ are given by

$$\begin{aligned} K &= (A\Sigma C' + BD')(C\Sigma C' + DD')^{-1} && (3) \\ \Sigma &= (A - KC)\Sigma(A - KC)' + BB' + KDD'K' - BD'K' - KDB'. && (4) \end{aligned}$$

From A, C and K, the optimal forecast of x_t is

$$\hat{x}_t = (A - KC)\,\hat{x}_{t-1} + Ky_t. \tag{5}$$

Under weak conditions Hansen and Sargent [2013] show that $(A - KC)$ is a stable matrix, so that this new representation writes as an infinite MA representation in terms of innovations. The measurement equation (2) rewrites as

$$y_t = C\hat{x}_{t-1} + u_t, \tag{6}$$

where the innovations vector u_t is then given by

$$u_t = C\left(x_{t-1} - \hat{x}_{t-1}\right) + D\varepsilon_t. \tag{7}$$

The innovations vector u_t is composed of two orthogonal components and the associated covariance matrix Σ_u is immediately deduced to be

$$\Sigma_u = C\Sigma C' + DD'. \tag{8}$$

Using a matrix decomposition such that $\Sigma_u = SS'$ (for example a Cholesky decomposition of Σ_u), it comes

$$I = S^{-1}C\Sigma C'S^{-1'} + S^{-1}DD'S^{-1'}. \tag{9}$$

Under the assumption that the factors perfectly account for the forecast errors of the state vector ,[5] i.e. $(x_{t-1} - \hat{x}_{t-1})$, the R_i^2 resulting from the linear projection of the i^{th} (standardized) residuals of the $AKC\Sigma$ representation on these factors is given by the (i, i) entries of

$$S^{-1}C\Sigma C'S^{-1'}(\equiv I - S^{-1}DD'S^{-1'}). \tag{10}$$

[4] If we consider a minimal ABCD form such as defined in Franchi and Paruolo [2015] the condition in Proposition 1 is necessary and sufficient. For non-minimal state-space systems, unstable eigenvalues of the $(A - BD^{-1}C)$ matrix still allow for a VAR representation in the observables provided a milder rank requirement is fulfilled (see Franchi and Paruolo [2015]).

[5] We will always maintain this assumption that it exists a set of relevant factors that perfectly reveal the state variables of the economy. See Stock and Watson [2016]

When the system is invertible, the observations on y_t perfectly forecast the state vector x_t and $\Sigma = 0$. It follows immediately that all the R_i^2 are zero. The matrix norm $\|S^{-1}C\Sigma C'S^{-1'}\|$ is a measure of the nonfundamentalness problem for the whole system.[6] This measure for the whole system differs from the one proposed by Soccorsi [2016] which is instead based on distance in covariances between the true nonfundamental innovations and innovations for its unique Wold representation. We can also look at each R_i^2 and thus isolate which type of shocks is more or less subject to nonfundamentalness and thus obtain a population measure for a specific shock in the same spirit that the one develops by Forni, Gambetti, and Sala [2016].[7] However, our contribution is to show that the R^2 diagnostic can be implemented on observable data under the assumption that factors well capture the state variables of the economy without conditioning on a specific model.[8] In fact, the implementation of the R^2 diagnostic on observable data gives a measure of the severity of the nonfundamentalness problem and the lag truncation bias. In this sense, the R^2 diagnostic is a broader measure of the finite VAR misspecification. The empirical R^2 can then be considered as an upper bound measure of the nonfundamentalness problem.[9]

1.2 A Lucas' Tree Model Example

We consider a simple formulation of the Lucas's tree model, with a single (unexpected or news) shock. The dividend of a tree, denoted a_t, is assumed to follow the process:

$$a_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1}, \tag{11}$$

where $\theta_0, \theta_1 \in [0, 1]$, $E(\varepsilon_t) = 0$ and $V(\varepsilon_t) = \sigma_\varepsilon^2$. Without loss of generality, we omit a relevant constant term in dividend for simplicity. Since we will consider later a univariate representation, we can normalize the variance of the shock to unity without loss of generality. In what follow, two polar cases are alternatively investigated: *i)* $\theta_0 = 1$ and $\theta_1 = 0$, in this case changes in dividends only result in an unexpected shock; *ii)* $\theta_0 = 0$ and $\theta_1 = 1$, dividends are governed by a news shock only.

[6] The matrix norm is defined here as $\|M\| = \sqrt{\text{trace}\,(M'M))}$.

[7] As in Soccorsi [2016] and Forni, Gambetti, and Sala [2016], we can also compute in our framework a measure which disentangle the part from the nonfundamentalness problem and the VAR lag truncation problem but it is not the object of the paper.

[8] It is crucial to understand that the nonfundamentalness measures proposed by Soccorsi [2016] and Forni, Gambetti, and Sala [2016] are only applicable in the context where the macroeconomic model is known. Consequently, those measure can not be readily adapted with observable data without knowing the true macroeconomic model.

[9] With a finite VAR of order p according to the state-space representation (1)–(2), one can show for the innovation vector u_t^p that $\Sigma_u^p = C\Sigma^p C' + DD'$ where $\Sigma^p \geq \Sigma$ in the matrix sense and Σ^p is the variance of the forecast error of the state variables x_{t-1} conditioning on $y_{t-1}, y_{t-2}, \dots, y_{t-p}$.

The representative consumer seeks to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i c_{t+i},$$

subject to the constraint

$$c_t + p_t n_{t+1} = (p_t + a_{t-1}) n_t \tag{12}$$

where E_t is the expectation operator conditional on the information set in period t, $\beta \in (0,1)$ is the subjective discount factor, c_t denotes consumption, p_t is the price of a tree and n_t is the number of trees held in period t. The supply of tree is one. The equilibrium asset price is given by $p_t = \beta E_t(p_{t+1} + a_t)$ and using the transversality condition we deduce the present value equation

$$p_t = \beta E_t \sum_{i=0}^{\infty} \beta^i a_{t+i}. \tag{13}$$

From the stochastic process (11), we obtain

$$E_t a_{t+1} = \theta_1 \varepsilon_t \quad \text{and} \quad E_t a_{t+i} = 0 \;\; \forall i > 1.$$

After replacement into (13), we get

$$p_t = \beta a_t + \beta^2 \theta_1 \varepsilon_t. \tag{14}$$

Using (11), we deduce a MA(1) process for the price

$$p_t = \beta(\theta_0 + \beta\theta_1)\varepsilon_t + \beta\theta_1 \varepsilon_{t-1}. \tag{15}$$

In terms of the $ABCD$ representation, equations (11) and (15) rewrite

$$x_t = \begin{pmatrix} 0 & \theta_1 \\ 0 & 0 \end{pmatrix} x_{t-1} + \begin{pmatrix} \theta_0 \\ 1 \end{pmatrix} \varepsilon_t \tag{16}$$

and

$$y_t = (0, \beta\theta_1)\, x_{t-1} + \beta(\theta_0 + \beta\theta_1)\varepsilon_t \;\;, \tag{17}$$

where $x_t = (a_t, \varepsilon_t)'$ and $y_t = p_t$. Here, we assume that the econometrician only observes the price p_t, but does not observe the dividends a_t. This assumption will simplify our computation, while keeping the main idea.

Let us first consider $\theta_0 = 1$ and $\theta_1 = 0$. In terms of $ABCD$, we have

$$A - BD^{-1}C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{18}$$

so the two eigenvalues are zero. In this case, there is no information problem (fundamental case) for the econometrician and the observed price perfectly reveals the state of the economy, i.e. $p_t = \beta a_t \equiv \beta\varepsilon_t$.

Second, we consider the case where $\theta_0 = 0$ and $\theta_1 = 1$. In terms of $ABCD$, we obtain

$$A - BD^{-1}C = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{\beta} \end{pmatrix} \tag{19}$$

In this case, the eigenvalues are 0 and $-1/\beta$. For $\beta < 1$, one eigenvalue exceeds unity in modulus and the representation is nonfundamental. Using only the price p_t, the econometrician cannot perfectly uncover the true state of the economy. The question is now to investigate how the severity of the nonfundamentalness problem varies with the value of β. Intuitively, small values of β makes the problem more severe as the modulus of the largest root is well above unity. Conversely, for $\beta \to 1$, the modulus of the largest root is close to unity, which mitigates the information problem for the econometrician. We now connect the eigenvalue (or β) to our simple R^2 diagnostic, when the econometrician performs the linear regression of the residual onto the past relevant variables (factors lagged once, i.e. $(x_{t-1} - \hat{x}_{t-1})$) and then compute the R^2 associated to that regression. If she obtains a large R^2, this indicates that the residuals of the regression are not orthogonal to past realization of the factor. Thus, the information problem may be severe and the identification of shocks be seriously biased. In the following proposition, we connect the value of β to the measurement error on the state variables and the R^2 diagnostic in the $AKC\Sigma$ setup.

Proposition 2 *The R^2 of the projection of u_t (as defined in equation (7)) on the lagged forecast errors of the state variables, i.e. $(x_{t-1} - \hat{x}_{t-1})$, is a decreasing concave function of β and is given by*

$$R^2 = 1 - \beta^2.$$

With news shock ($\theta_0 = 0, \theta_1 = 1$), the process is always nonfundamental as long as $\beta < 1$. We see from Proposition 2 a clear link between the severity of nonfundamentalness (the distance between the unstable root $1/\beta$ and unity) and the R^2 diagnostic. When β is small, the unstable root is large in modulus and the R^2 close to unity. This means in this simple example that the R^2 provides useful information about nonfundamentalness. Conversely, for $\beta \to 1$, the modulus of the unstable root tends to unity and the R^2 tends to zero. Notice that β measure the anticipation rate (see Ljungqvist and Sargent [2004] and Mertens and Ravn [2010]), i.e. the amount of which future information about future dividends are incorporated in today prices. If β is small (or close to zero), the current price contains little future information and this reflects into the high value (close to one) of the R^2

diagnostic. Conversely, if $\beta \to 1$, current prices do contain almost all the useful information about future dividends, and the R^2 is close to zero.

2 Quantitative Experiments and Simulation Results

In this section, we quantitatively illustrate how the R^2 defined section 1.1 can be a useful guide for VAR modeling. We first revisit the DSGE examples of Sims [2012]. We then next perform Monte Carlo simulations in finite sample using the Lucas' tree model of section 1.2. Finally, we investigate the relevance of our R^2 diagnostic from Monte Carlo simulations in finite sample using again the Lucas' tree model of section 1.2 and the DSGE models of section 2.1 in a bivariate SVAR setup.

2.1 Small Scale DSGE Models

We use the DSGE models of Sims [2012] to assess the reliability of SVARs. More precisely, Sims considers four model versions. The first, labeled "Full Model" features both real (under the form of habit formation and investment adjustment costs) and nominal (price rigidity) frictions. A second version ("Sticky Price") shuts down real frictions, but includes price rigidity. A third version ("RBC") is a frictionless representation of the economy. These three models are hit by two shocks: an unexpected shock and a news shock on TFP with an anticipation lag of three periods (meaning that the news comes three periods before it is implemented). A fourth version is the full model but with one period anticipation lag for the news shock. For the first three versions, Sims [2012] shows that the the "poor man's invertibility" condition is not satisfied when the econometrician observes TFP growth and (the log of) output (in deviation from stochastic trend). Conversely, with one lag in the news shocks, he finds that condition is satisfied. Even though the condition for invertibility is not satisfied for the first three models, Sims shows that SVARs yield reliable impulse responses for the structural shocks. We want to relate this result with our R^2 measure. To do so, we use the state-space representation and the parametrization as reported in the Appendix of Sims [2012] and we compute the corresponding $ABCD$ representation. The R^2 is directly obtained from the $ABCD/AKC\Sigma$ representation and the equation (10) in Section 1.1. We also compute the modulus of the largest eigenvalue of the matrix $(A - BD^{-1}C)$, that we denote $|\lambda|$. Table 1 reports $|\lambda|$ when it is above one and the R^2 associated to the unexpected and news shock on TFP. As the table makes clear, there exists a strong relationship between the modulus of largest eigenvalue and the R^2. For example, in the case of the full model for which the problem of noninvertibility is the most severe, the R^2 are 0.08 and 0.14. Conversely, the RBC model yields a modulus of largest eigenvalue close to unity and the R^2 become very close to zero. Note also that in the case of a one period anticipation, the condition for invertibility are satisfied and the R^2 are zero. The small value of the R^2 diagnostic coincides with the accuracy of SVAR impulse response functions as reported in Sims [2012].

Table 1: Modulus of the Largest Eigenvalue above one $|\lambda|$ and R^2

Model	Modulus of the largest eigenvalue above one	R^2 Unexpected shock	R^2 News shock
Full	1.32	0.08	0.14
RBC	1.06	0.04	0.00
Sticky Price	1.24	0.08	0.01
Full, One Period News	-	0	0

Notes: The models are the ones presented in Sims [2012]. The full version is a DSGE model with both real and nominal frictions. The RBC model is a version with no real and nominal frictions. The sticky price model is a version without real friction. For these three versions, the news comes three periods in advance.

We also explore the sensitivity of the modulus of the largest eigenvalue and the R^2 to the discount factor β. We do that in the frictionless RBC model of Sims [2012]. As in the Lucas' tree model, high values (close to one) for β put more weights on future expectations and thus help the econometrician to better identify news shocks. We consider of values for β in the interval $[0.95, 0.999]$. Results are reported in Figure 1. Panel (a) shows that the modulus of the largest eigenvalue of $A - BD^{-1}C$ monotonically decreases as β increases. Panel (b) reports the relationship between this eigenvalue and the R^2 (for the news shock). As β increases, the R^2 goes down as the nonfundamentalness problem is les severe. These quantitative results extend the analytical results of Proposition 2, that were obtained from the Lucas' tree model.

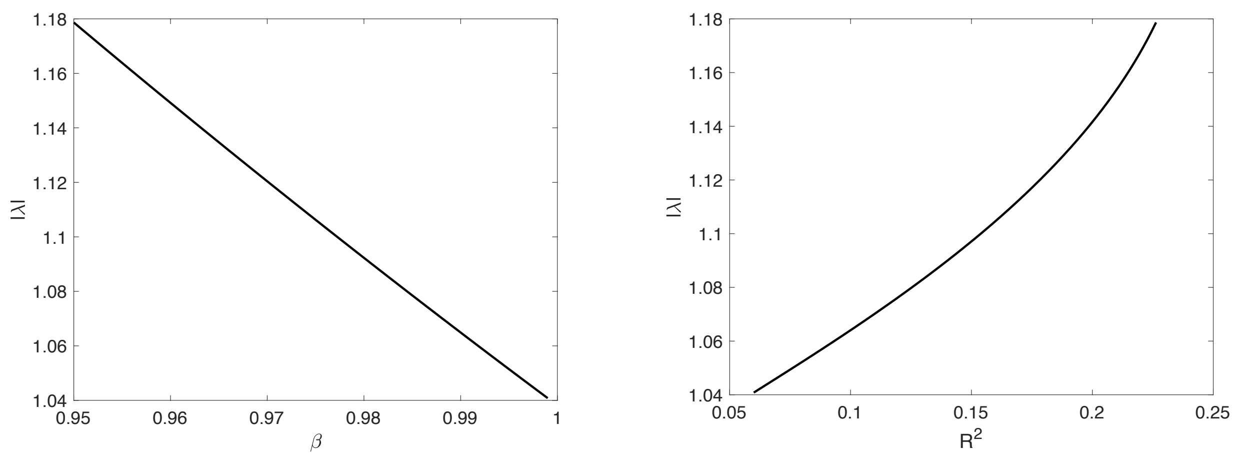
2.2 Simulation Results in a Univariate Setup

We now evaluate the reliability of our R^2 diagnostic and the previous analytical results of section 1.2 using simulation experiments. In our Monte–Carlo study, we generate 1000 data samples from the Lucas' tree model of section 1.2 (equations (11) and (14)). Every data sample consists of 200 observations and corresponds to the typical sample size of empirical studies. In order to reduce the effect of initial conditions, the simulated samples include 500 initial points which are subsequently discarded in the estimation.

Figure 1: Discount Factor (β), Modulus of the Largest Eigenvalue $|\lambda|$ and R^2

(a) $|\lambda|$ vs β

(b) $|\lambda|$ vs R^2: news shock



Notes: The model used as the Data Generating Process is the RBC model of Sims [2012]. $|\lambda|$ is the modulus of the largest eigenvalue of $A - BD^{-1}C$

The econometrician observes the price p_t and estimates am autoregressive process of order q

$$p_t = \sum_{i=1}^{q} \gamma_i p_{t-i} + \nu_t \tag{20}$$

and then estimates by OLS the autoregressive parameters as well as the standard–error of the residuals. We then compare the Impulse Response Function (IRF) from the estimations and with true ones that are obtained from equation (15), and compute the cumulative Root Mean Square Errors (RMSE) for a given horizon.[10] The RMSE accounts for both bias and dispersion of the estimated IRFs.

We vary the model's parameters[11] and compute the modulus of the largest eigenvalue of $(A - BD^{-1}C)$, $|\lambda|$, which is between 0.1 and 1.9. We also normalize the standard deviation of the shock such that the response of the price p_t to the shock ε_t on impact is one.

We also assume that the econometrician can estimate a factor $\tilde{s}_t$ (or possibly more than one) than spans the true state of the economy. This factor is obtained from a panel of N times series. We assume that the practitioner observes

$$s_{i,t} = \varepsilon_t + \alpha \zeta_{i,t} \quad i = 1, \cdots, N \quad , \tag{21}$$

where $\zeta_{i,t}$ is a measurement error. The $\zeta_{i,t}$ were produced according to a multivariate autoregressive process

$$\zeta_{i,t} = M\zeta_{i,t-1} + \tilde{\zeta}_{i,t} \quad , \tag{22}$$

with $M = \rho I_N$ and the scalar $|\rho| \leq 1$. The $\tilde{\zeta}_{i,t}$ are Gaussian white noises (orthogonal to the structural shock ε_t at all leads and lags) with a covariance matrix $\Sigma_{\tilde{\zeta}}$ given by

$$\Sigma_{\tilde{\zeta}} = \begin{pmatrix} 1 & \tau & \dots & & \dots & \tau \\ \tau & 1 & \tau & \dots & \dots & \tau \\ \tau & \tau & \ddots & & & \vdots \\ \vdots & & & \ddots & & \vdots \\ \vdots & & & & \ddots & \tau \\ \tau & \dots & & \dots & \tau & 1 \end{pmatrix} ,$$

with $|\tau| \leq 1$. We adopt this particular representation for the M (same autoregressive parameter ρ) and $\Sigma_{\tilde{\zeta}}$ (same correlation structure) matrices to maintain parsimony in the parametrization of our

[10] This measure is defined as $\text{crmse}(k) = \sum_{i=0}^{k} \text{rmse}_i$ where k denotes the selected horizon, $\text{rmse}_i = ((1/1000)\sum_{j=1}^{1000}(\text{irf}_i(\text{model}) - \text{irf}_i(\text{svar})^j)^2)^{1/2}$ the RMSE at horizon i, $\text{irf}_i(\text{model})$ the true impulse response function of p_t given by equation (15) and $\text{irf}_i(\text{svar})^j$ the impulse responses function of p_t from autoregressions for the j^{th} draw.

[11] More precisely, we set $\beta = 0.5$ and $\theta_1 = 1$ and we select a grid of values for θ_0 between 0.026 and 9.5.

possible large panel. Following Forni and Gambetti [2014], each $\tilde{\zeta}_{i,t}$ ($i = 1, ..., N$) is associated to a random standard deviation σ_i uniformly distributed between 0 and 1. In addition, the parameter α can vary to control for the size of the measurement errors. For example, if α is close to zero, the estimated factor almost perfectly coincides with ε_t.

In our benchmark simulations, the econometrician estimates the factor $\tilde{s}_t$ as the first principal component of $\{s_{i,t}\}$. The number of lags q in (20) is equal to six for all our experiments. The R^2 diagnostic is obtained in the following way. We estimate equation (20) and recover estimated residuals $\widehat{\nu}_t$. These residuals are linearly projected on the current and lagged estimated factor $\tilde{s}_t$. The cumulative RMSE is computed for a horizon $k = 5$, since after 3 periods the true response is zero. Finally, the panel of $s_{i,t}$ contains $N = 20$ series.[12]

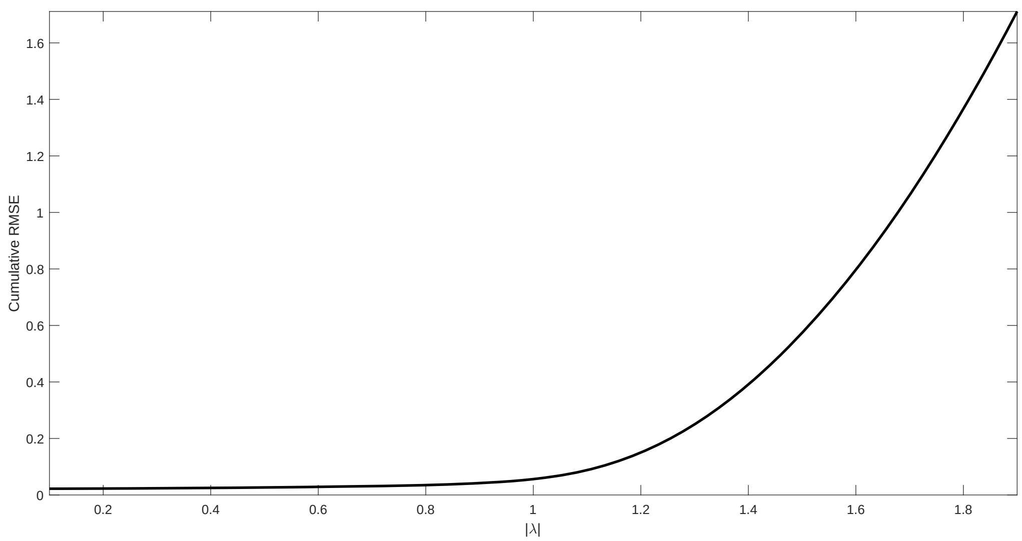
Our benchmark results are obtained for $\alpha = 0.001$, so the correlation between the true state and the first principal component is almost one. We will further assess the reliability of our R^2 diagnostic when α increases. Let the $(A - BD^{-1}C)$ largest eigenvalue in modulus $|\lambda|$ varies between 0.1 and 1.9 and compute the cumulative RMSE to evaluate the accuracy of regression (20). As Figure 2-(a) makes clear, the cumulative RMSE is small and stable when $|\lambda| < 1$. As $|\lambda|$ becomes larger than one (the nonfundamental representation) and increases, the cumulative RMSE continuously grows up: nonfundamentalness clearly deteriorates the reliability of the autoregressive representation. We know turn to Figure 2-(b) that reports our R^2 diagnostic with respect to $|\lambda|$. When $|\lambda| < 1$, the R^2 is very close to zero (and stable), meaning that the observations of the current and lagged values of p_t allow to properly identify the effect of the shock. As $|\lambda|$ exceeds unity, the R^2 continuously increases with $|\lambda|$. Therefore, the R^2 is a good indicator of the accuracy of autoregressions. It displays a very similar pattern as the one of the cumulative RMSE: it is flat for $|\lambda| < 1$ and progressively increases when $|\lambda| > 1$. As such, the R^2 is almost a sufficient statistic to assess the reliability of autoregressions. Finally, note that we do not see any strong variations in the neighborhood of $|\lambda| = 1$, which confirms that nonfundamentlaness is not always a quantitatively relevant feature.

We also use our simulations to assess Proposition 2 in finite sample. Figure 3 reports the results. As this figure shows, Proposition 2 is highly supported by our finite sample experiments. The estimated R^2 tends to over-estimate the theoretical one near and above the unstable root of the MA representation. However, these two R^2 are closer and closer as the unstable root increases. Moreover, when we include a very large number of lags in the autoregression ($q = 30$), the two figures also perfectly coincides (see Figure 10 in Appendix B). With a large number of lags, the

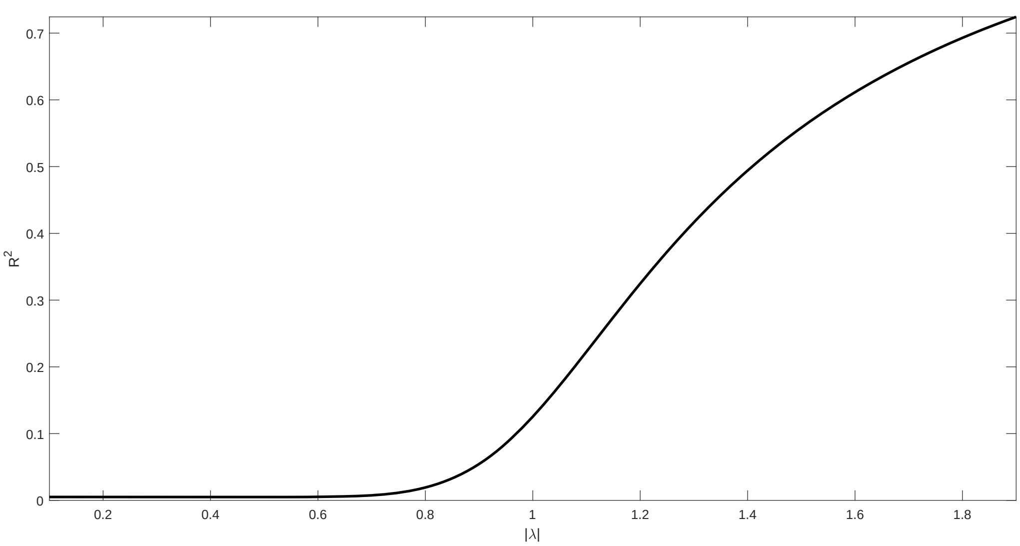
[12]We have also investigated larger panels without altering our findings.

Figure 2: Cumulative RMSE, R^2 and Nonfundamentalness

(a) $|\lambda|$ vs Cumulative RMSE



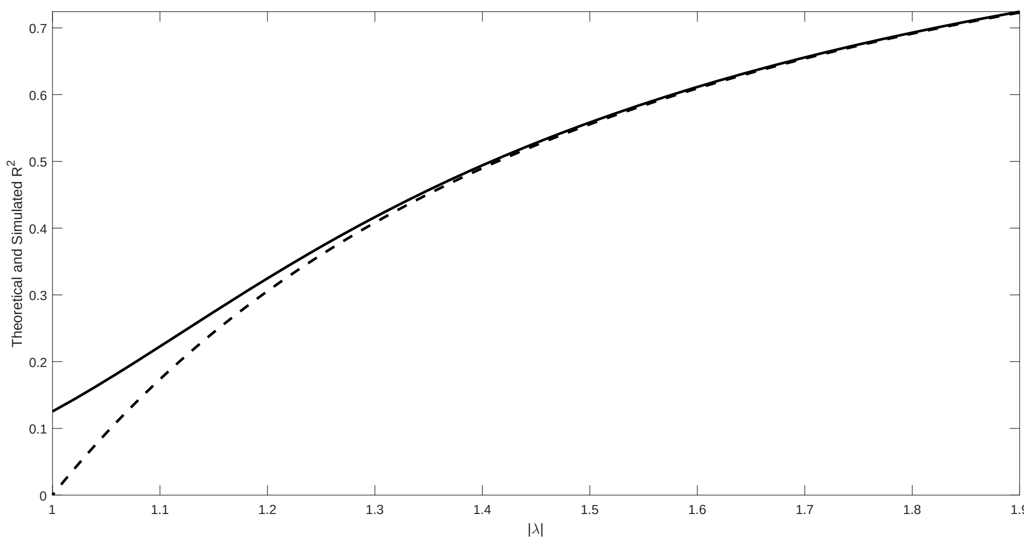
(b) $|\lambda|$ vs R^2



Notes: The two panels of this Figure are obtained from the simulations of the Lucas' tree model (equations (11) and (14)) of section 1.2. We vary parameter θ_0 such that the model solution maximum eigenvalue in modulus $|\lambda|$ varies between .1 and 1.9. For each θ_0, the model is simulated 1000 times and each simulation is 200 observations long. Equation (20) cumulative RMSE (panel (a)) and model's R^2 diagnostic (panel (b)) are then plot against $|\lambda|$, the modulus of the largest eigenvalue of $A - BD^{-1}C$.

autoregressive model better capture the moving average representation, but the model with $q = 6$ lags delivers very reliable results.

Figure 3: Theoretical R^2 and Simulated One in Finite Sample



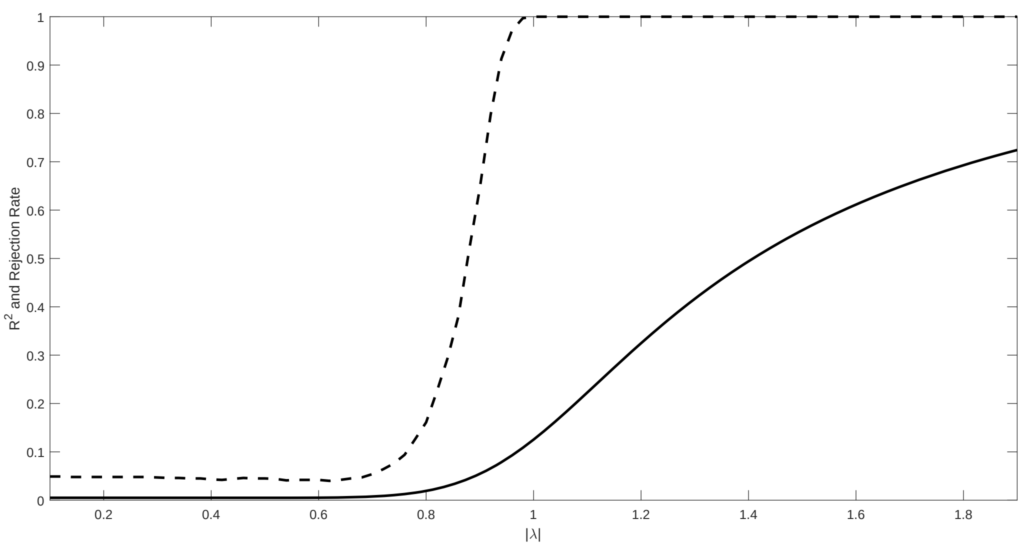
Notes: This Figure is obtained from the simulations of the Lucas' tree model (equations (11) and (14)) of section 1.2. We vary parameter θ_0 such that the model solution maximum eigenvalue in modulus $|\lambda|$ varies between 1 and 1.9. For each θ_0, the model is simulated 1000 times and each simulation is 200 observations long. The solid line corresponds to the R^2 diagnostic obtained from simulations and the dashed line to the theoretical R^2 of Proposition 2.

In our setup, in order to obtain some indications about a threshold R^2 above which the autoregression (20) is unreliable, we perform the following computation. We select the threshold R^2 such that the true impulse response function after one period lies within the 95% confidence interval of the estimated responses. In our benchmark case, this yields a value around 0.24 . Notice that autoregressions still deliver reliable results when $|\lambda| > 1$, as our threshold $R^2 = 0.24$ is associated with $|\lambda| = 1.12$. Obviously, this threshold R^2 can not be used as a general result as it depends on many factors (the setup, the number of variables, the selected confidence interval). However, we will find similar threshold using additional simulation experiments in a bivariate setup (see the next section) and VARs estimated on actual US data in section 4.

We also compare our R^2 diagnostic to the orthogonality test proposed by Forni and Gambetti [2014], which is a test of nonfundamentalness.[13] To do so, we report in Figure 4 the rejection rate of the test statistics (at 5%) with the R^2. We obtain that the orthogonality test appears very

[13] See Proposition 3 in Section 3.3 for an exposition and the related discussion.

Figure 4: R^2 and the Rejection Rate of Forni and Gambetti's [2014] Test



Notes: This Figure is obtained from the simulations of the Lucas' tree model (equations (11) and (14)) of section 1.2. We vary parameter θ_0 such that the model solution maximum eigenvalue in modulus $|\lambda|$ varies between .1 and 1.9. For each θ_0, the model is simulated 1000 times and each simulation is 200 observations long. The solid line corresponds to the R^2 diagnostic and the dashed line to the rejection rate of the the orthogonality test proposed by Forni and Gambetti [2014].

sensitive to the parametrization, as it over-rejects the null hypothesis in the fundamental case (when the eigenvalue is within the range $(0.7, 1)$). For example, when the eigenvalue is 0.89, the R^2 is equal to 5% whereas the rejection rate is around 58%. Moreover, the threshold R^2 (around 0.25) below which autoregressions yield accurate IRFs is associated to a 100% rejection rate.[14] Because it smoothly varies in the neighborhood of $|\lambda| = 1$, the R^2 seems to be, in finite sample, a better measure of nonfundamentalness severity than the orthogonality test.

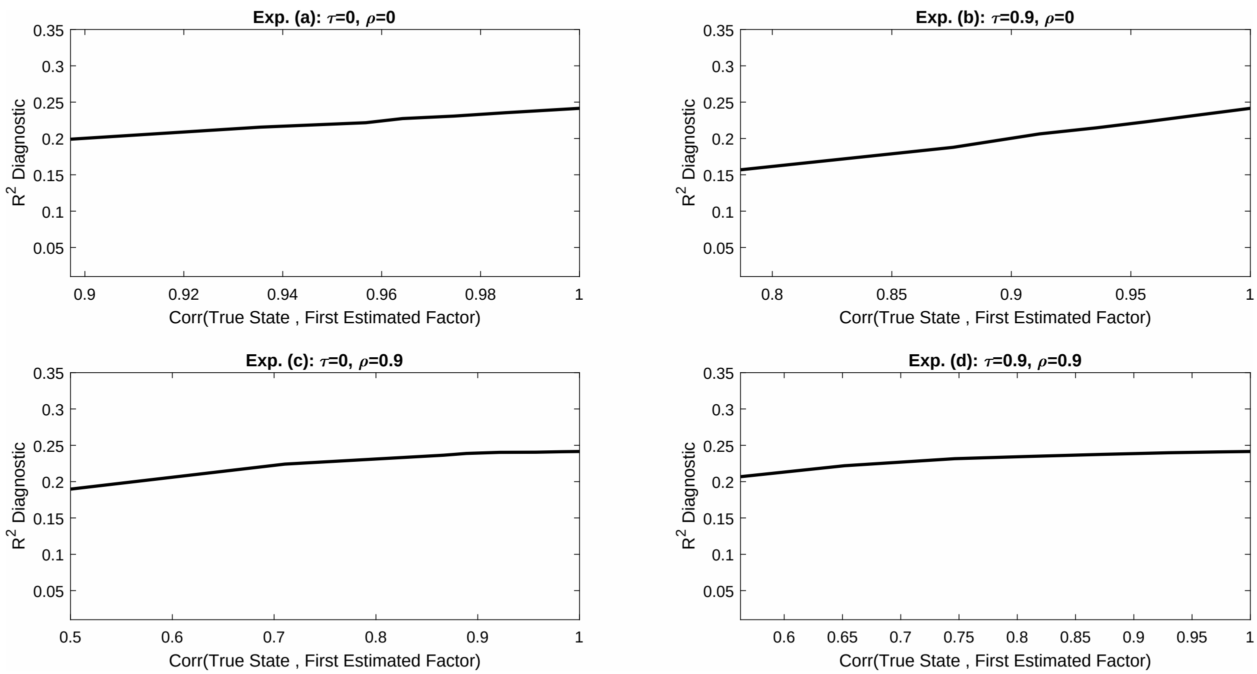
Now, we investigate the robustness of our previous findings to the case of N times series (See the discussion above about equation (21)). We conduct four experiments (See Exp. (a) to Exp. (d) in Figure 5). First, we consider a panel of N uncorrelated (and serially uncorrelated) measurement errors where $\tau = \rho = 0$. Second, we correlate these measurement errors ($\tau = 0.9$) without serial correlation ($\rho = 0$). Third, we consider serial correlation ($\rho = 0.9$), but measurement errors are mutually uncorrelated ($\tau = 0$). Finally, we set $\tau = \rho = 0.9$, so that measurement errors both display mutual correlation and serial dependance. We modify the value of α, such that the measurement errors $\zeta_{i,t}$ account more and more for the variance of the N variables $s_{i,t}$. Consequently, the first principal component of $s_{i,t}$ ($i = 1, ..., N$) is less and less a proper measure of the state of the economy. To measure the reliability of the estimated factor to span the true state of the economy, we compute the correlation[15] between ε_t (the true state of the economy) and the first principal component $\tilde{s}_t$ for different values of α.

At the same time, we compute the threshold R^2 such that the true impulse response lies in the 95% confidence interval. As Figure 5 shows, in the four case we consider (See Exp. (a) to (d) in Figure 5), our R^2 diagnostic is only slightly affected, as it appears almost flat regarding the change in the parameter α (inversely reflecting by the correlation between the true state and the estimated one, or the contribution of the first factor to the panel.). This shows that a threshold value for the R^2 diagnostic around 0.25 seems to be good reference for accuracy in that context. Moreover, the R^2 sensitivity can be reduced by increasing the number of principal components used. When we use the first four principal components isntead of the first one only, we obtain a threshold value for the R^2 that is slightly higher (around 0.30), but less polluted by measurement errors and unsensitive to the different alterations of the benchmark case. Our simulation experiments then confirm the usefulness of our R^2 diagnostic to assess the reliability of small-scale autoregressions.

[14]When the R^2 is equal to 0.15 we already get a 100% rejection rate.

[15]Another measure is related to the contribution of the first factor to the set of N variables. We obtain the same results, as the the contribution of the first component decreases (as our R^2 diagnostic) when the value of α increases. We thank a referee for pointing out this quantitative issue.

Figure 5: R^2 when the Econometrician Observes the State with Noise



Notes: This Figure is obtained from the simulations of the Lucas' tree model (equations (11) and (14)) of section 1.2. We vary parameter α and for each α, the model is simulated 1000 times and each simulation is 200 observations long. For each simulation, we compute the correlation between the true state of the model and the estimated first factor, which is directly a function of α. We then compute our proposed R^2 with the first estimated factor.

2.3 Simulation Results in a Bivariate SVAR Setup

Let us repeat the above analysis when the DGP contains two shocks, so that a bivariate SVAR should be used to estimate IRFs. We first consider a slight modification of the Lucas' tree model that includes both unexpected and news shocks to dividends. Second, we investigate a RBC model with a similar stochastic structure. For both experiments, we maintain the sample sample size, initial conditions and number of simulations as in Section 2.2. We also inspect the orthogonality test proposed by Forni and Gambetti [2014] and we obtain a strong rejection rate.

Lucas' tree model with news and surprise on dividends: We first consider the Lucas' tree model presented in section 1.2, except that the process of a is now a random walk and has a news component ε and a surprise one u:

$$a_t = a_{t-1} + \varepsilon_{t-2} + u_t,$$

where ε_t and u_t are gaussian with unit variance.[16] In that case, the equilibrium price p of a tree is given by

$$p_t = \frac{\beta}{1-\beta} a_t + \frac{\beta}{1-\beta} \left(\beta \varepsilon_t + \varepsilon_{t-1}\right).$$

The structural moving average representation of the solution is given by

$$\begin{pmatrix} \Delta a_t \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} L^2 & 1 \\ \frac{\beta^2}{1-\beta} + \beta L & \frac{\beta}{1-\beta} \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ u_t \end{pmatrix}. \tag{23}$$

The determinant of the moving average coefficients matrix in (23) is $-\frac{\beta^2}{1-\beta} - \beta L + \frac{\beta}{1-\beta} L^2$. The roots of that determinant are 1 and $-\beta$. Therefore, if we assume that only current and past values of a and p are observed by the econometrician, the shocks ε_t and u_t are nonfundamental for the variables Δa_t and Δp_t, and cannot be recovered by an econometrician, because $|\beta| < 1$. The basic setup of Beaudry and Portier's [2006] is therefore always non-fundamental. However, by varying the discount factor β, we can evaluate the severity of the problem. We therefore use the model as the DGP and simulate data. The news shock is then identified as the shock that has no effect on impact on dividends in a $(\Delta a_t, \Delta p_t)$ Vector Error Correcting Model with one cointegrating relation.[17] We then compare the estimated IRF of the asset price p to a news shock, and perform the Forni and Gambetti's [2014] sufficient information test for nonfundamentalness. In our context,

[16]Our results are robust to a change of the dividend process (longer or shorter news) and to the relative size of the variances.

[17]See Beaudry and Portier [2006] and Beaudry and Portier [2014] for a discussion. We will consider in more details this identification scheme in the statistical setup of Section 3.

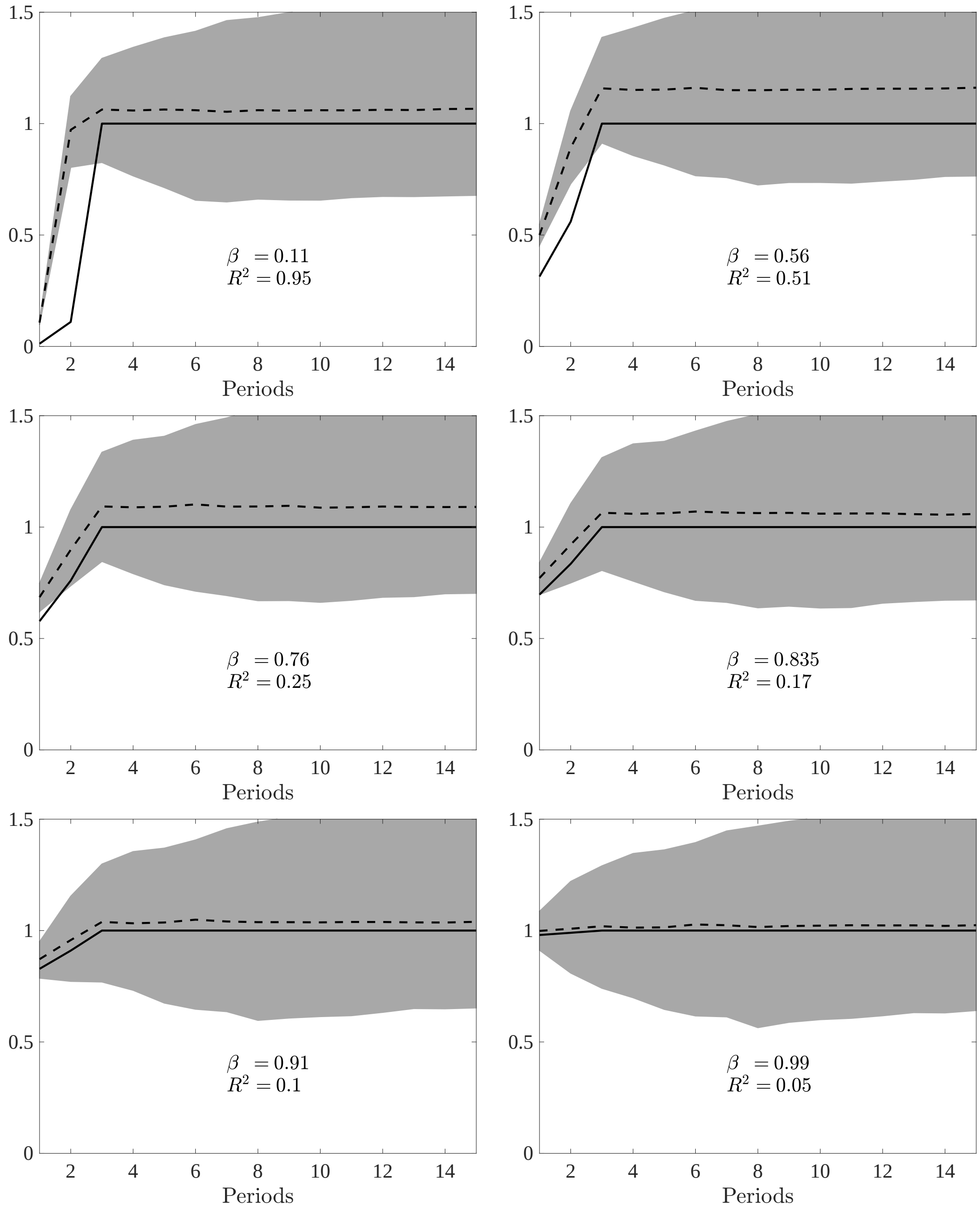
the test consists in projecting the estimated news shock on a constant and the past of the true ε_t and u_t that have been used to generate the data. As the shocks are *i.i.d.* and orthogonal one to each other, the R^2 of that regression should be 0 if the estimated shock was indeed equal to ε_t. Figure 6 (from (a) to (f)) reports the results of our Monte-Carlo experiments, when we vary the discount factor β from 0.1 to 0.99 (so the unstable root from 10 to almost 1). The figure also includes our R^2 diagnostic. The figure clearly shows that when β is small (future values of dividends does not matter a lot), the accuracy of the SVAR model is small, as the estimated IRF is outside the confidence interval. However, as β increases the SVAR model uncovers more and more the true response. As in the univariate case, it appears that SVARs yields proper estimates when the R^2 diagnostic is around 0.25.

RBC model with news and surprise on TFP: As a final Monte-Carlo investigation, consider the RBC model of Sims [2012] that we used in section 2.1. Assume TFP is a random walk with a surprise and a three-period ahead news shock. Notice that in this simple RBC framework, the response of GDP is negative in the short run[18]. The VAR model includes TFP growth and the (log of) de-trended output (from the non-stationary TFP). The VAR is estimated with 8 lags, as in Sims [2012].[19] As in the previous case, news shocks are identified using a zero restriction on impact on TFP. Figure 7 reports the results of our experiments for the response of output to a news TFP shock. In this figure, we consider different values of the discount factor β. Notice that with 3 lags in the news shock, the RBC model always admits a non-fundamental representation in the associated (TFP growth, De-trended Output) VAR representation, but the severity of nonfundamentalness varies with the discount factor as emphasized in the two previous sub-sections (See the reported value of $|\lambda|$ in each figure). As discussed above, lowering the discount rate increases the largest eigenvalue and lowers the weight on forward-looking component, rendering the nonfundamentalness problem more severe. For low values of β (around 0.95), the accuracy of SVARs is small as the bivariate model faces difficulty to properly reproduce the responses of output to a news TFP shock in the short run. However, when we increase β, the SVAR model better uncovers the true responses. This finding immediately echoes the results reported in Figure 1 obtained from the $ABCD/AKC\Sigma$ setups. For example, when $\beta = 0.985$, the SVAR model produces a confidence interval for IRFs that includes the true one. We again obtain that a value for the R^2 around 0.25 is

[18] Despite controversies about the empirical response of output to news TFP shocks (See Beaudry and Portier [2014] for a discussion), this framework remains useful for our quantitative analysis, as we just want to investigate the reliability of our R^2 diagnostic (See Barsky and Sims [2011] and Sims [2012] for a similar quantitative investigation).

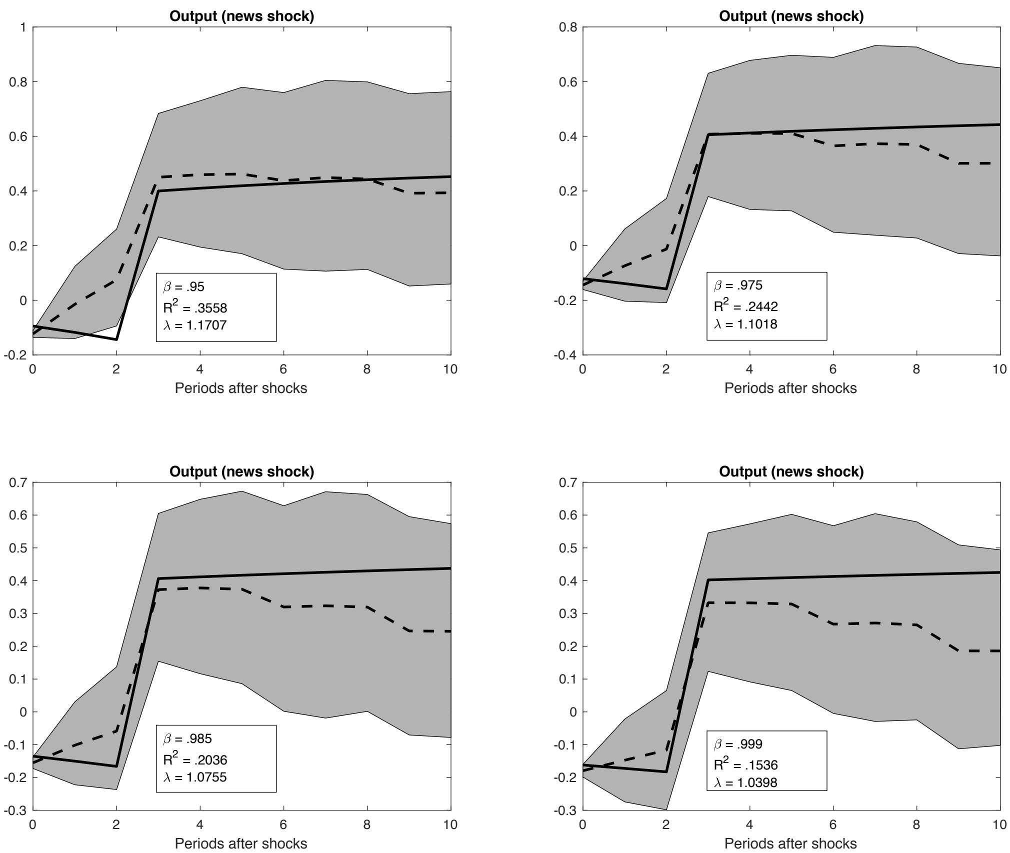
[19] We investigate the sensitivity of the results to other lags selection. When the number of lags is reduced, the SVAR model uncovers less likely the true IRFs.

Figure 6: Response of the Asset Price to the Identified News and R^2 when Using the Lucas' Tree Model as the DGP



Notes: For this Figure, the Lucas' tree model with two shocks of section 1.2 has been simulated 1000 times over 250 period for various levels of β. A dimension two Vector Error Correcting model with five lags and one cointegrating relation has been estimated using simulated dividends a and asset price p. The Figure shows the response to a shock that does not affect dividends a on impact, as estimated using the VECM (dashed line for the mean response, 95% confidence band for the grey area), together with the theoretical response to a news shock ε. This Figure includes the setting value for β and the associated R^2.

Figure 7: Response of Output to the Identified News and R^2 when Using the RBC Model as the DGP



Notes: This Figure is obtained from the simulations of the RBC model of Section 2.1. We vary parameter β and for each β, the model is simulated 1000 times and each simulation is 250 observations long. For each simulation, we compute the correlation between the estimated shock and the true state vector of the model. The solid line corresponds to the true IRF, the dashed line to the estimated (from the SVAR model) IRFs. Grey areas correspond to the 95% confidence band. This Figure includes the setting value for β, the associated R^2 and the largest eigenvalue in modulus$|\lambda|$ of $(A - BD^{-1}C)$.

associated with a very accurate estimation of the true IRFs from SVARs.

For these two simulation experiments, we perform the orthogonality test proposed by Forni and Gambetti [2014]. We obtain a strong rejection rate for all the cases we investigated, even when the SVAR model properly uncovers the true IRFs. This result provides additional evidences against the usefulness of this test and favors our simple R^2 diagnostic.[20]

3 The R^2 Diagnostic in the SVAR Setup

The previous section has set the scene by showing the relevance of our R^2 diagnostic in order to assess the consequences of nonfundamentalness in the estimation context of VAR models. In this section, we formally make the connection. We could consider a factor model as in Forni, Giannone, Lippi, and Reichlin [2009] but as argued by Forni and Gambetti [2014] the most natural solution is to extend the VAR by adding principal components corresponding to a FAVAR model. We then recast the state-space representation (1) and (2) as a FAVAR model by considering that the factors are a proper approximation of the true state variables. We investigate a misspecified VAR representation that would omit the factors and thus not fully capture dynamics of the state variables. Using that misspecified model to identify structural shocks, we show that the bias in recovering these shocks is of the size of the R^2. The R^2 is obtained from the projection of (misspecified) structural shocks on the past of the factors orthogonal to the lagged values of the variables included in the VAR, which corresponds to testing the orthogonality of the estimated structural shock with respect to the lags of the factors (Forni and Gambetti [2014]).

3.1 The Econometric Setup

Assume the well-specified model labeled as $\mathcal{M}_0$ corresponding to the following FAVAR:

$$\begin{aligned} y_t &= B_y(L)y_{t-1} + B_f(L)f_{t-1} + \epsilon_{yt}, \\ f_t &= C_f(L)f_{t-1} + \epsilon_{ft}, \end{aligned} \qquad (\mathcal{M}_0)$$

where the vector y_t contains n variables of interest and f_t is a vector of (observed) relevant q factors and $B_y(L)$, $B_f(L)$ and $C_f(L)$ are matrices of finite polynomials in lag operator. We suppose that the variance of each factor is normalized to unity.[21] Our goal is to assess the quantitative effects of omitting the relevant set of factors f_t at the estimation stage and thus when identifying the true

[20] See the previous Section 2.2 and the Corollary 1.

[21] This representation adds factors in a VAR representation of the data and thus differs from a more general representation of dynamic factor models (see Stock and Watson [2005] and [2016]).

structural shocks. The analysis is then similar to a standard omitting variable problem in linear regression.

Now define the vector of Wold innovations $\epsilon_t = \left(\epsilon'_{yt} \;\; \epsilon'_{ft}\right)'$ and $\epsilon_t = A\eta_t$ where η_t is the vector of the structural shocks. The dimension of the vector η_t can be different than the dimension of the vector ϵ_t. Assume that there is a unique linear transformation that maps innovations $\epsilon_{y,t}$ into the subset of structural shocks of interest $\eta_{y,t}$ according to

$$\epsilon_{yt} = A_0 \eta_{yt}, \tag{24}$$

where A_0 is a nonsingular matrix. We then consider here the square case, namely, the number of structural shocks of interest η_{yt} equals the dimension of the vector ϵ_{yt}. As usual, we impose the normalization assumption that $E(\eta_{yt}\eta'_{yt}) = I_n$. This orthogonality/normalization assumption is not sufficient (except if $n = 1$) to identify the structural shocks, since $E(\epsilon_{yt}\epsilon'_{yt}) = A_0 A'_0$ is symmetric. At least $n(n-1)/2$ restrictions must be imposed to identify A_0. To fix ideas, we assume that identification is achieved by imposing point restrictions in the form

$$\Gamma \text{ vec}\,(A_0) = \gamma, \tag{25}$$

where Γ is a $(m \times n^2)$ selection matrix and γ a $(m \times 1)$ vector of m restrictions greater or equal to $n(n-1)/2$. These restrictions can be generalized to other identification schemes. Combining with the covariance matrix, these additional restrictions allow to identify each elements of A_0. $\gamma = 0$ corresponds to a case of zero impact restriction, which is one of the most commonly used in the SVAR literature (see Ramey [2016]). In what follows, we do not need to be explicit about the identifying restrictions Γ, and will keep the matrix A_0 unspecified.

3.2 The Misspecified Model

Now, suppose that the econometrician estimates the following misspecified VAR model $\mathcal{M}_1$:

$$y_t = \tilde{B}_y(L) y_{t-1} + \tilde{\epsilon}_{yt}, \tag{$\mathcal{M}_1$}$$

where $\tilde{B}_y(L)$ is a matrix of finite polynomials of lag operator . We further assume that the econometrician uses the restrictions (25) to identify the structural shocks.

This model improperly ignores the role played by the factors f_t. We are in a typical case of missing relevant variables in VARs.[22] The omitted variables problem will affect the misspecified

[22]See Stock and Watson [2001], [2005], Canova [2006], Canova [2007] and Lütkepohl [2005] for a discussion of this issue.

VAR model $\mathcal{M}_1$ in various ways. First, by omitting the lagged factors, the VAR model will not properly uncover the size of the shocks, because part of the identified structural shocks will be polluted by the missing lagged factors. Second, the omitting of lagged factors will affect the dynamics of y_t and the polynomial matrices $\tilde{B}_y(L)$ do not properly summarize the true dynamic structure of the observables y_t. Third, the covariance structure of the variables y_t and f_t can affect the proper measurement of the auto-regressive matrices $\tilde{B}_y(L)$ at the estimation stage. In what follows, we explicitly measure the bias of the identified structural shocks by omitting the factors.

The restricted structural shocks (the ones obtained from $\mathcal{M}_1$) are denoted $\tilde{\epsilon}_{yt} = \tilde{A}_0\tilde{\eta}_{y,t}$. We impose the same normalization assumption $E(\tilde{\eta}_{y,t}\tilde{\eta}_{y,t}') = I_n$ and the same additional restrictions

$$\Gamma \text{ vec}\left(\tilde{A}_0\right) = \gamma, \tag{26}$$

where Γ and γ are the same as in model $\mathcal{M}_0$. Denoting $\tilde{\Sigma} = E(\tilde{\epsilon}_{yt}\tilde{\epsilon}_{yt}')$ and $\Sigma = E(\epsilon_{yt}\epsilon_{yt}')$, we deduce $\tilde{A}_0\tilde{A}_0' = \tilde{\Sigma} \geq \Sigma = A_0A_0'$ in the matrix sense and $\|\tilde{A}_0\| \geq \|A_0\|$, because the error term $\tilde{\epsilon}_{y,t}$ omits the factor f_{t-1}. We now examine in more details the effects of omitting the factors in the estimation of model $\mathcal{M}_1$ and for the identification of structural shocks.

3.3 Testing for Nonfundamentalness

Testing for nonfundamentalness for a particular structural shock is achieved by performing an orthogonality test of the estimated structural shock $\tilde{\eta}_{it}$ with respect to the lags of the factors as proposed by Forni and Gambetti [2014]. As previously discussed in the introduction, the testing procedure implies that the econometrician observes the factors.

Proposition 3 *For a given sample of size T, the following relation holds between the Wald statistics W_T of the orthogonality test and the R_i^2 of the projection of the (misspecified) structural shocks $\tilde{\eta}_{it}$ on the lags of the factors orthogonal to y_{t-1}:*

$$W_T = T\frac{R_i^2}{(1-R_i^2)}.$$

The Wald statistic is composed of two terms. The first term T (the size of the sample) refers to the precision of the estimation, since the covariance matrix of the factors has been normalized to identity. The second term R_i^2 accounts for the explanatory power of the lagged factors for the (misspecified) structural shocks $\tilde{\eta}_{it}$.

Corollary 1 *Suppose that the Wald statistics is greater than its critical value, so that the test rejects the fundamentalness of the residuals (or of identified structural shocks from the wrong* *model). Such a a rejection is compatible with arbitrarily low level of the R_i^2, and therefore with little quantitative importance on the nonfundamentalness problem, as long as the sample size T is large enough.*

Let us illustrate Corollary 1. Consider a single factor ($q = 1$) in the regression and a sample of size $T = 200$, as very usual in applied time series macroeconomics. In this case, the limiting distribution of the Wald statistic under the null hypothesis that the identified shock $\tilde{\eta}_{it}$ is orthogonal to the lagged factor is a chi-square statistic with one degree of freedom. Its critical value at 5% is 3.84. This implies an associated critical R^2 equals to 0.0192. In words, it is possible to reject fundamentalness even though the lagged factor explains only less than 2% of the variance of the identified structural shock.[23]

Proposition 4 formalizes the relationship between the R^2 of the projection of (misspecified) structural residuals and the distance to the true model.

Proposition 4 *The R_i^2 statistic is :*

(i) a consistent estimator of the distance between the misspecified structural shock $\tilde{\eta}_{it}$ and the true structural shock $\eta_{it} = e_i'\eta_{yt}$. Indeed, when R_i^2 is small, $\tilde{\eta}_{it}$ is close to η_{it}. Moreover, when $R_i^2 = 0$ for all $i = 1, \ldots, n$, $\|\tilde{A}_0 - A_0\| = 0$.[24]

ii) a consistent estimator of the distance between the misspecified impulse responses of i-th misspecified structural shock $\tilde{\eta}_{it}$ and the true impulse responses of the i-th structural shock η_{it}.

(iii) a consistent estimator of the distance between the misspecified variance decomposition on impact and the true one.

The meaning of Proposition 4 is that if R^2 is small for a specific (misspecified) structural shock, the distance between the well-specified model $\mathcal{M}_0$ and the misspecified model $\mathcal{M}_1$ for this shock is small even if the Wald test rejects fundamentalness. In this sense, the R^2 is a measure of incomplete parametric encompassing of the model $\mathcal{M}_0$ by the model $\mathcal{M}_1$ as defined by Mizon and Richard [1986].

3.4 Characterization of Biases on Impact

As a simple characterization of biases, we use a bivariate SVAR model. Anticipating our empirical results (see section 4), we consider the benchmark VAR model of Beaudry and Portier [2006]

[23] In Section 2.2 and 2.3, we have already shown from our Monte-Carlo experiments in finite sample that the R^2 is a better diagnostic of nonfundamentalness than the orthogonality test.

[24] The matrix norm is defined as in footnote 6.

with a short run restriction to identify a news shock on technology. The results expounded here are not specific to their setup. As discussed in Ramey [2016], this triangular representation introduced by Sims [1980] (using a Cholesky decomposition) is the most commonly used identification method in macroeconomics (for example, for monetary policy or government spending shocks). The following results thus apply to many other SVARs.

The bivariate model of Beaudry and Portier [2006] includes Total Factor Productivity (TFP) and a measure of Stock Prices (SP). The technological news $\eta_{2,t}$ is the shock that is orthogonal to current TFP. The well-specified model is $\mathcal{M}_0$ with $y_t = (TFP_t, SP_t)'$ and other relevant factors, while the econometrician is estimating $\mathcal{M}_1$ with only these two variables. According to the structural assumption, A_0 is lower triangular and given by

$$A_0 = \begin{bmatrix} a_{0,11} & 0 \\ a_{0,21} & a_{0,22} \end{bmatrix}. \tag{27}$$

Under the misspecified model $\mathcal{M}_1$, we maintain the same identifying restriction, such that the misspecified impact matrix $\tilde{A}_0$ is given by

$$\tilde{A}_0 = \begin{bmatrix} \tilde{a}_{0,11} & 0 \\ \tilde{a}_{0,21} & \tilde{a}_{0,22} \end{bmatrix}. \tag{28}$$

Using the same logic than before, we can derive proposition 5.

Proposition 5 *For small R_1^2 and R_2^2 , the relative biases of the impact response for the identified structural shocks satisfy:*

$$\begin{aligned} \frac{\widehat{\tilde{a}}_{0,11} - a_{0,11}}{a_{0,11}} &\simeq \frac{1}{2} R_1^2, \\ \frac{\widehat{\tilde{a}}_{0,22} - a_{0,22}}{a_{0,22}} &\leq \frac{1}{2} R_2^2. \end{aligned}$$

The proposition characterizes the relative biases for the estimated impact responses of the TFP to an unexpected technology shock and the stock prices to a news shock. In particular, the relative bias impact response of the stock price to a news shock is smaller than half of the R_2^2. This proposition makes explicit that from a quantitative point of view, it is not the value of the Wald statistics but the size of the R^2 that matters for the bias caused by nonfundamentalness.

4 Application to the Identification of TFP News Shocks with U.S. Data

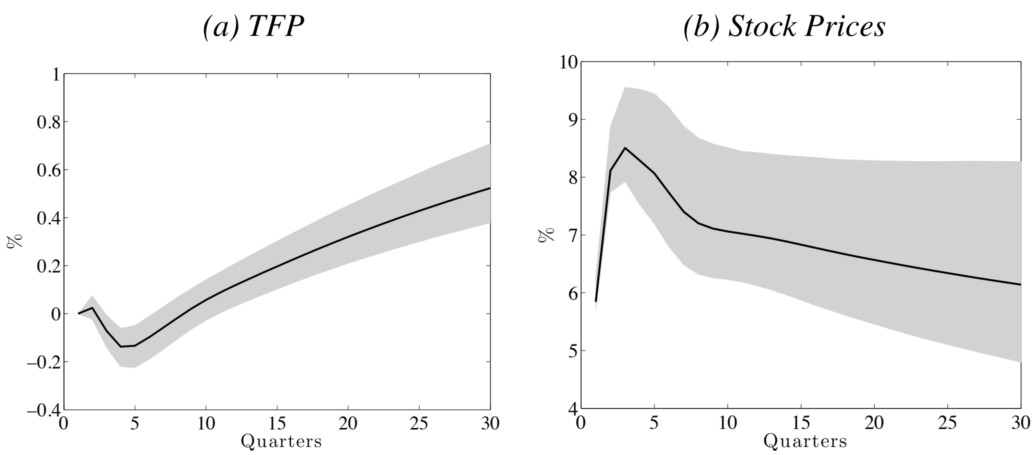
As an illustration, we apply the R^2 diagnostic to the results of Beaudry and Portier [2006] and [2014].

4.1 Baseline Results

In the following, we use the same sample as used by Forni, Gambetti, and Sala [2014] and use the data described in Beaudry and Portier [2014]. Note that the results of our VARs are robust to a longer sample (1946-2013), but the factors are only available on the shorter sample. TFP is corrected for utilisation, consumption is total consumption (including durable) and investment is total investment (see the data appendix F).

We first consider the basic Beaudry and Portier's [2006] bivariate VAR. Whereas the small dimension of the VAR might be a weakness, this VAR has the advantage of being simple and, as discussed in Beaudry and Portier [2014], gives results that are robust to various extensions. The two variables in the system are TFP and Stock Prices. The single identifying restriction is that the identified news has no impact effect on TFP, which correspond to a Choleski decomposition in which TFP is the first variable and the news shock the second shock. Figure 8 shows that we indeed identify a diffusion news. TFP does not increase for about 10 quarters [25], but does in the long run.

Figure 8: Response to a News Shock in the Beaudry and Portier's [2006] VAR 2



Notes: Data are described in the appendix and the sample period in 1960Q1-2012Q2. The news shock is the one that does not affect TFP on impact. The VAR include 4 lags. The unit of the vertical axis is percentage deviation from the situation without shock. Grey areas correspond to the 66% confidence band. The distribution of IRF is the Bayesian simulated distribution obtained by Monte-Carlo integration with 10,000 replications, using the approach for just-identified systems discussed in Doan [1992].

We now extend the VAR to add three extra variables: consumption, investment and hours. To identify a TFP news shock, we follow the identification strategy set out in Beaudry and Portier

[25] TFP actually decreases, which might be the consequence of an excessive correction for utilization.

[2014] which is a natural extension to that introduced in Beaudry and Portier [2006]. This identification strategy identifies two technology shocks as the only ones that are allowed affect TFP in the long run. One of them is unrestricted, while the other one is constrained not to affect TFP on impact. That second shock is potentially a technology news shock. The other shocks in the VAR remain unnamed. The identifying restrictions is therefore: *(i)* all the shocks but the unrestricted technology shock have zero impact effect on TFP, *(ii)* the news and the unrestricted technology shock are the only permanent shocks to TFP. In Beaudry and Portier [2014] it is shown that this identification gives robust results when one varies either the information set, the sample period and the specification.

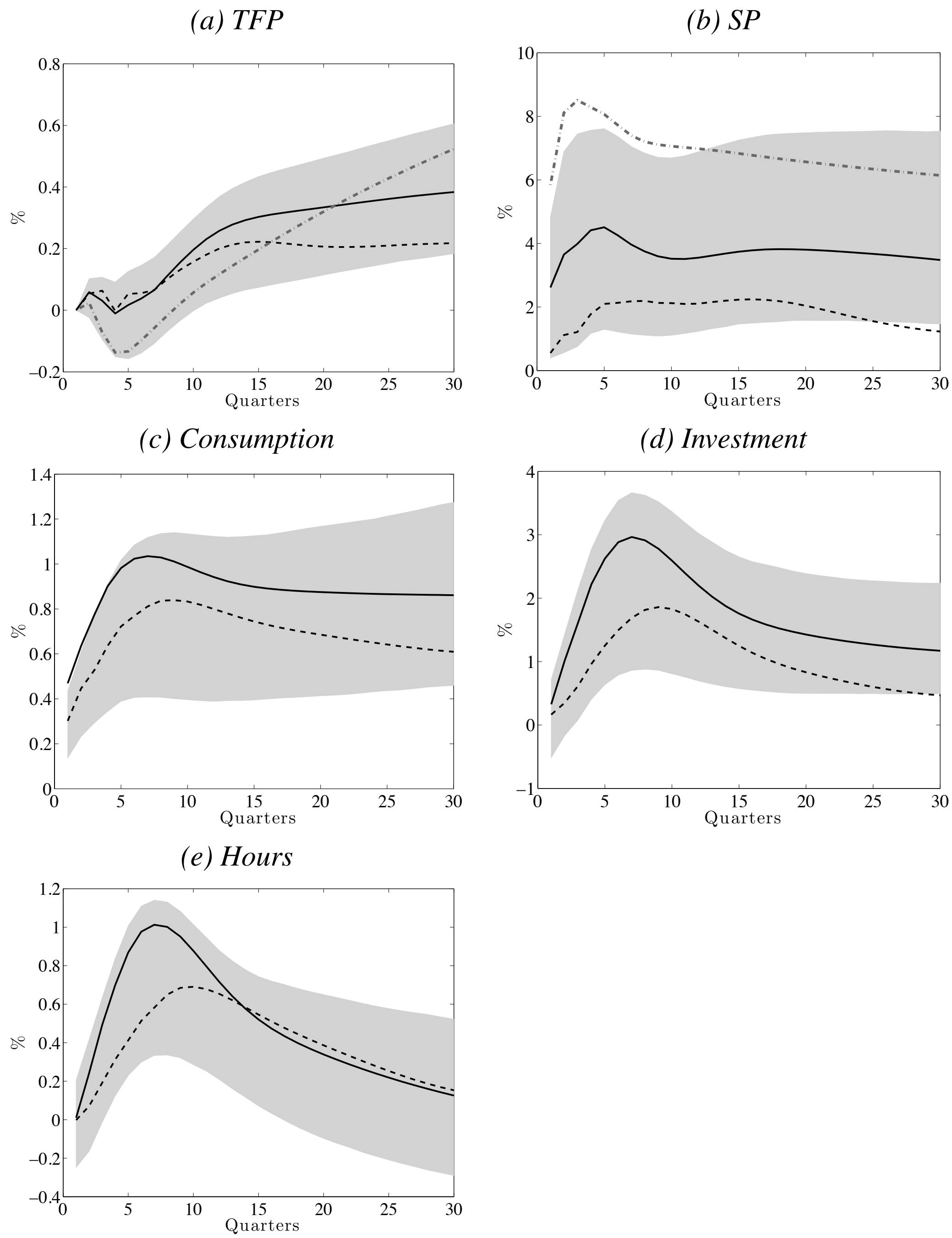
Impulse responses are presented in Figure 9. The plain line shows the point estimates. We observe all the characteristic of a news driven economic expansion. TFP does not move in the short run, the stock market reacts instantaneously to the news, consumption, investment and hours do increase on impact and subsequently, before any sizable increase in TFP. In panels (a) and (b), we also represent the responses of TFP and SP obtained from the VAR 2 (dashed-dotted gray line). Note that the response of TFP is very similar, while the response of SP is now purged from some non-news related variations.

These results suggest that there are indeed news in the business cycle, but it might be the case, as pointed out by Forni, Gambetti, and Sala [2014], that the estimation suffers from nonfundamentalness. This is what we check now.

4.2 The Quantitative Unimportance of Nonfundamentalness

In order to test for nonfundamentalness, we follow Forni, Gambetti, and Sala [2014]. The authors use a dataset composed of 107 US quarterly macroeconomic series, and estimate the principal components of this data set. They show that essentially all the information is contained in the first three factors. We therefore use these first three factors. We project the estimated news shock of the VAR 2 and of the VAR 5 on one lag or four lags of the first three factors, and test for the orthogonality of our news shocks to the factors. The Wald statistics and the p-values are reported in Table 2. In all cases, the p-value is less that 5%. We therefore do agree with Forni, Gambetti, and Sala [2014] that our identified strategy is likely subject to the nonfundamentalness problem. However, does it matter for the estimation of the impulse response functions to a news shock? The answer is no, or at least not very much. A first element suggestive of this negative answer comes from the inspection of the R^2 associated with specification test. These are displayed in Table 2. The R^2 are never larger than .2: even though our estimated news shocks are not orthogonal

Figure 9: Comparison of the VAR 5 Responses With the Ones of the VAR 5 Augmented With the First Three Factors



Notes: Data are described in the appendix and the sample period is 1960Q1-2012Q2. In the VAR 5 (the plain line), the news shock is restricted to have no impact effect on TFP but is not restricted in the long run. The dotted lines correspond to the VAR 8, that is the VAR 5 augmented with the first three factors of Forni, Gambetti, and Sala [2014]. The dashed-dotted gray lines of panels (a) and (b) are the responses to a news shock in the VAR 2 of Figure 8. The VARs include 4 lags. The unit of the vertical axis is percentage deviation from the situation without shock. Grey areas correspond to the 66% confidence band of the VAR 5. The distribution of IRF is the Bayesian simulated distribution obtained by Monte-Carlo integration with 10,000 replications, using the approach for just-identified systems discussed in Doan [1992].

Table 2: Test for Nonfundamentalness and Associated R^2s

Model	One lag		Four lags	
	R^2	F-test p-value	R^2	F-test p-value
VAR 2	.03	.04	.18	.05
VAR 5	.09	.01	.21	.01

Notes: This Table presents the results of the sufficient information test proposed by Forni and Gambetti [2014]. For each VAR, the news shock is projected on one or four lags of the first three factors of Forni, Gambetti, and Sala [2014]. Table includes the p-value for the orthogonality test, as well as the R^2 of those regressions. Data are described in the appendix and the sample period in 1960Q1-2012Q2. In the VAR 2, the news shock is the one that does not affect TFP on impact.In the VAR 5 , the news shock is only restricted to have no impact effect on TFP but is not restricted in the long run. The VARs are estimated in levels and with 4 lags.

to the factors, those factors explain less than 20% of the variance of the news. The theoretical results and the simulation experiments of the previous section suggest that in such a case, the nonfundamentalness should not be much of a quantitative problem.[26]

We then re-estimate our VAR 5 by adding the three factors, so that we end up estimating a VAR 8. We use the same identification strategy, that is, : *(i)* all the shocks but the unrestricted technology shock have zero impact effect on TFP, *(ii)* the news and the unrestricted technology shock are the only permanent shocks to TFP. The estimated responses to the newly identified news shock are the black dashed lines of Figure 9. Except for the Stock Price whose response has a similar shape but is divided by a factor two, the responses of TFP, consumption, investment and hours are all very similar to that obtained in the absence of including the factors. This contrasts with Forni, Gambetti, and Sala [2014] finding that is based on the identification strategy of Barsky and Sims [2011], which itself is not very supportive of the news shocks view of business cycles. Hence, these results suggests that our chosen means of identifying news shocks generate impulse responses with properties that are robust to the nonfundamentalness critique. There may remain debate about how best to identify new shocks, but that is an issue entirely different form the issue of nonfundamentalness emphasized in Forni, Gambetti, and Sala [2014].[27] We therefore infer that nonfundamentalness is not likely an important factor in evaluating whether or not news shocks are relevant for business cycles.

[26]See also Beaudry, Fève, Guay, and Portier [2015]

[27]See Beaudry, Nam, and Wang [2011] for some answers to that question.

5 Conclusion

In this paper, we have proposed a simple R^2 diagnostic to asses the severity of nonfundamentalness in SVARs. Building on the $ABCD$ and $AKC\Sigma$ setups, we have shown how the R^2 allows to measure the distance between the structural models and SVARs. Using a simple Lucas's tree model with news shock, we have connected the R^2 with the discount factor. We have also performed simulation experiments to highlight how the R^2 can be a useful guide for VAR modelling. We then have developed a FAVAR setup and characterized the bias when factors are omitted. We have notably shown that the relative bias in recovering the true structural shocks is of the order of half the R^2 of the projection of the misspecified structural shocks on the true ones. An application to news shock with US data indicates that nonfundamentalness is present, but it does not appear to matter quantitatively (the R^2 is small). This is not of course a general result that would apply to all SVARs exercises. In fact, the test proposed by Forni and Gambetti [2014] is a useful one that macro-econometricians should systematically perform when nonfundamentalness may be present. However, this test should be accompanied with the computation of the R^2 associated to the test, in order to assess whether nonfundamentalness is likely to be quantitative important.

References

BARSKY, R. B., AND E. R. SIMS (2011): “News shocks and business cycles,” *Journal of Monetary Economics*, 58(3), 273–289.

BEAUDRY, P., P. FÈVE, A. GUAY, AND F. PORTIER (2015): “When is Nonfundamentalness in VARs a Real Problem? An Application to News Shocks,” NBER Working Papers 21466, National Bureau of Economic Research, Inc.

BEAUDRY, P., D. NAM, AND J. WANG (2011): “Do Mood Swings Drive Business Cycles and is it Rational?,” NBER Working Papers 17651, National Bureau of Economic Research, Inc.

BEAUDRY, P., AND F. PORTIER (2006): “Stock Prices, News, and Economic Fluctuations,” *American Economic Review*, 96(4), 1293–1307.

——— (2014): “News Driven Business Cycles: Insights and Challenges,” *Journal of Economic Literature*, 52(4).

BLANCHARD, O. J., AND D. QUAH (1989): “The Dynamic Effects of Aggregate Demand and Supply Disturbances,” *American Economic Review*, 79(4), 655–73.

CANOVA, F. (2006): “You can use VARs for structural analyses. A comment to VARs and the Great Moderation,” mimeo, Universitat Pompeu Fabra.

——— (2007): *Methods for Applied Macroeconomic Research*, Economics Books. Princeton University Press.

CANOVA, F., AND M. HAMIDI SAHNEH (2016): “Are Small-Scale SVARs Useful for Business Cycle Analysis? Revisiting Non-Fundamentalness,” mimeo, BI Norvegian Business School and Universidad Carlos III, forthcoming in *Journal of European Economic Association*.

CHEN, B., J. CHOI, AND J. ESCANCIANO (2012): “Testing for Fundamental Vector Moving Average Representation,” Dp, Indiana University.

DOAN, T. (1992): *Rats Manual*. Estima, Evanston, IL.

FERNÁNDEZ-VILLAVERDE, J., J. F. RUBIO-RAMÍREZ, T. J. SARGENT, AND M. W. WATSON (2007): “ABCs (and Ds) of Understanding VARs,” *American Economic Review*, 97(3), 1021–1026.

FORNI, M., AND L. GAMBETTI (2014): “Sufficient information in structural VARs,” *Journal of Monetary Economics*, 66(C), 124–136.

FORNI, M., L. GAMBETTI, AND L. SALA (2014): “No News in Business Cycles,” *The Economic Journal*, 124(581), 1168–1191.

——— (2016): “VAR Information and the Empirical Validation of DSGE Models,” Discussion Papers 11178, CEPR, forthcoming in *Journal of Applied Econometrics*.

FORNI, M., D. GIANNONE, M. LIPPI, AND L. REICHLIN (2009): “Opening The Black Box: Structural Factor Models With Large Cross Sections,” *Econometric Theory*, 25(05), 1319–1347.

FRANCHI, M., AND P. PARUOLO (2015): "Minimality of State Space Solutions of DSGE Models and Existence Conditions for Their VAR Representation," *Computational Economics*, 46(4), 613–626.

HAMIDI SAHNEH, M. (2015): "Are the shocks obtained from SVAR fundamental?," Mpra paper no. 65126, Universidad Carlos III.

HANSEN, L. P., AND T. J. SARGENT (2013): *Recursive Models of Dynamic Linear Economies*, no. 10141 in Economics Books. Princeton University Press.

KILIAN, L., AND H. LÜTKEPOHL (2017): *Structural Vector Autoregressive Analysis*. Cambridge University Press.

LIPPI, M., AND L. REICHLIN (1993): "The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment," *American Economic Review*, 83(3), 644–52.

——— (1994): "VAR analysis, nonfundamental representations, blaschke matrices," *Journal of Econometrics*, 63(1), 307–325.

LJUNGQVIST, L., AND T. J. SARGENT (2004): *Recursive Macroeconomic Theory*. The MIT Press, Cambridge, Mass., 2nd edn.

LÜTKEPOHL, H. (2005): *New Introduction to Multiple Time Series Analysis*. Springer.

MERTENS, K., AND M. RAVN (2010): "Measuring the Impact of Fiscal Policy in the Face of Anticipation: A Structural VAR Approach," *Economic Journal*, 120(544), 393–413.

MIZON, G. E., AND J.-F. RICHARD (1986): "The Encompassing Principle and Its Application to Testing Non-nested Hypotheses," *Econometrica*, 54(3), 657–678.

RAMEY, V. (2016): *Macroeconomic Shocks and Their Propagation*vol. 2 of *Handbook of Macroeconomics*, chap. 2, pp. 71–162. Elsevier.

SIMS, C. A. (1980): "Macroeconomics and Reality," *Econometrica*, 48(1), 1–48.

SIMS, E. R. (2012): "News, Non-Invertibility, and Structural VARs," in *Advances in Econometrics. DSGE Models in Macroeconomics: Estimation, Evaluation, and New Developments*, ed. by N. Balke, F. Canova, F. Milani, and M. A. Wynne, vol. 28. Emerald Group Publishing Limited,.

SOCCORSI, S. (2016): "Measuring nonfundamentalness for structural VARs," *Journal of Economic Dynamics and Control*, 71(C), 86–101.

STOCK, J., AND M. WATSON (2001): "Vector Autoregressions," *Journal of Economic Perspectives*, 15(4), 101–116.

——— (2005): "Implications of Dynamic Factor Models for VAR Analysis," NBER Working Papers 11467, National Bureau of Economic Research, Inc.

——— (2016): *Dynamic Factor Models, Factor-Augmented Vector Autoregressions, and Structural Vector Autoregressions in Macroeconomics*vol. 2 of *Handbook of Macroeconomics*, chap. 8, pp. 415–525. Elsevier.

Appendix

A Proof of Proposition 2

Using A, B, C and D, we have to solve for the Kalman gain K and the matrix Σ of forecast errors:

$$K = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \equiv (A\Sigma C' + BD')(C\Sigma C' + DD')^{-1}$$

and

$$\Sigma = \begin{pmatrix} V(a_t - \hat{a}_t) & Cov(a_t - \hat{a}_t, \varepsilon_t - \hat{\varepsilon}_t) \\ Cov(\varepsilon_t - \hat{\varepsilon}_t, a_t - \hat{a}_t) & V(\varepsilon_t - \hat{\varepsilon}_t) \end{pmatrix} \equiv \begin{pmatrix} \sigma_{aa} & \sigma_{ae} \\ \sigma_{ea} & \sigma_{ee} \end{pmatrix}$$

Using the innovation representation, the matrix Σ is given by:

$$\Sigma = (A - KC)\Sigma(A - KC)' + BB' + KDD'K' - BD'K' - KDB'$$

Let us first consider the vector K. Using A, B, C and D and given Σ, we deduce

$$k_1 = \frac{\sigma_{ee}}{\beta(\sigma_{ee} + \beta^2)} \quad \text{and} \quad k_2 = \frac{1}{\sigma_{ee} + \beta^2}$$

Now, we insert K into the covariance matrix Σ and we use again A, B, C and D. We obtain

$$\begin{aligned} \sigma_{aa} &= \frac{\beta^4 \sigma_{ee}}{(\sigma_{ee} + \beta^2)^2} + \frac{\beta^2 \sigma_{ee}^2}{(\sigma_{ee} + \beta^2)^2} \\ \sigma_{ae} &= \sigma_{ea} = -\frac{\beta \sigma_{ee}}{(\sigma_{ee} + \beta^2)^2} \\ \sigma_{ee} &= \frac{\beta^2 \sigma_{ee}}{(\sigma_{ee} + \beta^2)^2} + 1 + \frac{\beta^4}{(\sigma_{ee} + \beta^2)^2} - \frac{2\beta^2}{\sigma_{ee} + \beta^2} \end{aligned}$$

σ_{aa} and σ_{ae} depends on σ_{ee}, whereas σ_{ee} can be solved independently. After some algebra, we obtain $\sigma_{ee} = 1 - \beta^2$ and we immediately deduce the covariance matrix of forecast errors:

$$\Sigma = (1 - \beta^2) \begin{pmatrix} \beta^2 & -\beta \\ -\beta & 1 \end{pmatrix}$$

and the Kalman gain K

$$K = \begin{pmatrix} \frac{1-\beta^2}{\beta} \\ 1 \end{pmatrix}$$

Now the variance of the residual u_t in the infinite order AR representation of the observed variable y_t is given by

$$\begin{aligned} \sigma_u^2 &= C\Sigma C' + DD' \\ &= \beta^2(1 - \beta^2) + \beta^4 \\ &= \beta^2 \end{aligned}$$

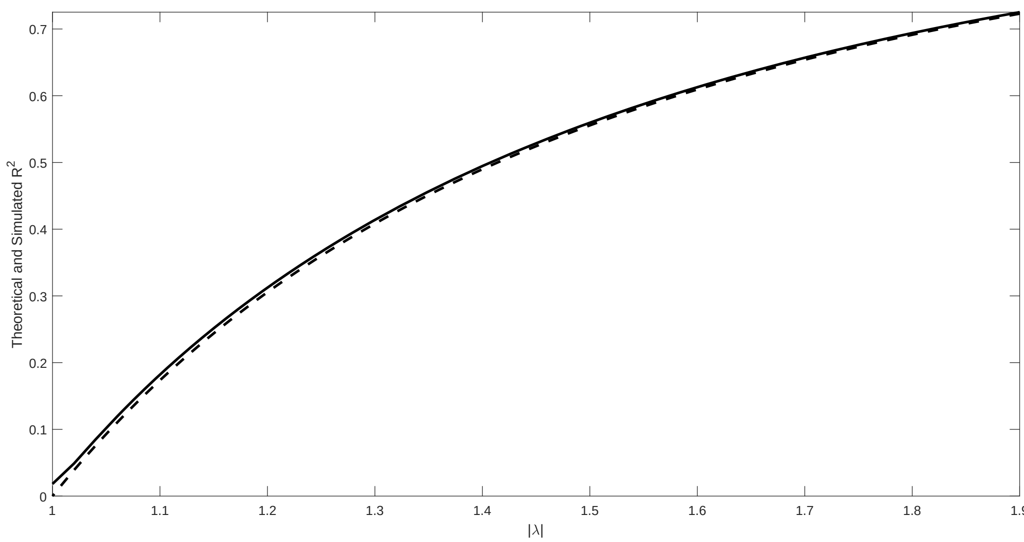
So the linear regression of u_t on $x_{t-1} - \hat{x}_{t-1}$ (or a factor that accurately represents the state vector) yields a coefficient of determination

$$R^2 = \frac{C\Sigma C'}{\sigma_u^2} \equiv 1 - \beta^2$$

This completes the proof.

B Additional Simulation Experiments

Figure 10: Theoretical R^2 and Simulated One in Finite Sample with a Large Number of Lags



Notes: This Figure are obtained from the simulations of the Lucas' tree model (equations (11) and (14)) of section 1.2. We vary parameter θ_0 such that the model solution maximum eigenvalue in modulus $|\lambda|$ varies between 1 and 1.9. For each θ_0, the model is simulated 1000 times and each simulation is 200 observations long. The solid line corresponds to the R^2 diagnostic obtained from simulations and the dashed line to the theoretical R^2 of Proposition 2.

C Proof of Proposition 3 :

To simplify the exposition of the proof, we consider a FAVAR model with one lag only. Results can be easily extended to a more general lags structure. Moreover any FAVAR(p) (or VAR(p)) can be recasted as a FAVAR(1) (or VAR(1)). We first characterize the relationship between the identified misspecified structural shock and the factors f_t. The vector of the residuals from the estimation of model $\mathcal{M}_1$ is given by

$$\widehat{\tilde{\epsilon}}_y = M_Y \tilde{\epsilon}_y = M_Y Y,$$

where $\tilde{\epsilon}_y$ is the $T \times n$ of error terms for each of the n equations, Y is the $T \times n$ matrix of the corresponding y_t and $M_Y = I - Y_{-1} \left(Y_{-1}' Y_{-1}\right)^{-1} Y_{-1}'$ is the orthogonal projection matrix to Y_{-1}, *i.e.* the matrix containing the lagged values of Y. Using model $\mathcal{M}_0$ and the same notations, we deduce

$$M_Y Y \quad = \quad M_Y F_{-1} B_f' + M_Y \epsilon_y,$$

where F_{-1} is a $T \times q$ matrix containing the lagged values of the factors. This implies for the estimated misspecified structural shocks

$$\widehat{\tilde{\eta}}_y \widehat{\tilde{A}_0}' = M_Y F_{-1} B_f' + M_Y \epsilon_y.$$

where the vector $\tilde{\eta}_y$ contains the elements $\tilde{\eta}_{yt}$. Since $\widehat{\tilde{\epsilon}}_{yt} = \tilde{\epsilon}_{yt} + o_p(1)$, $\widehat{\tilde{\eta}}_y = \tilde{\eta}_y + o_p(1)$ and $\widehat{\tilde{A}_0} = \tilde{A}_0 + o_p(1)$, we obtain [28]

$$\tilde{\eta}_y \widetilde{A}_0' = M_Y F_{-1} B_f' + M_Y \epsilon_y + o_p(1).$$

By $M_Y \epsilon_y = \epsilon_y + o_p(1)$, we can write

$$\tilde{\eta}_y = M_Y F_{-1} B_f' \left(\tilde{A}_0'\right)^{-1} + \epsilon_y \left(\tilde{A}_0'\right)^{-1} + o_p(1).$$

Using the linear relation between innovations $\epsilon_{y,t}$ and the structural shocks η_{yt}, this finally yields

$$\tilde{\eta}_y = M_Y F_{-1} B_f' \left(\tilde{A}_0'\right)^{-1} + \eta_y A_0 \left(\tilde{A}_0'\right)^{-1} + o_p(1).$$

Now suppose for the sake of exposition that we are interested in one specific structural shock η_{it}. By the above expression, one gets

$$\tilde{\eta}_{it} = e_i' \tilde{\eta}_{yt} \equiv e_i' \tilde{A}_0^{-1} B_f \hat{f}_{t-1} + e_i' \tilde{A}_0^{-1} A_0 \eta_{yt} + o_p(1), \tag{C.1}$$

with e_i a selecting vector that is composed of zeros and one at the ith element. The variable $\hat{f}_{t-1}$ is the orthogonal projection of the lags of the f_{t-1} onto the space generated by y_{t-1}, namely

$$\hat{f}_{t-1}' = f_{t-1}' - y_{t-1}' \left(\sum_{t=2}^{T} y_{t-1} y_{t-1}'\right)^{-1} \sum_{t=2}^{T} y_{t-1} f_{t-1}'.$$

We can rewrite equation (C.1) under the form

$$\tilde{\eta}_{it} = e_i' \tilde{\eta}_{yt} = \delta_i' \hat{f}_{t-1} + e_i' \tilde{A}_0^{-1} A_0 \eta_{yt} + o_p(1). \tag{C.2}$$

We now define $v_{it} = e_i' \tilde{A}_0^{-1} A_0 \eta_{yt}$. In a matrix form, equation (C.2) rewrites:

$$\tilde{\eta}_i = M_Y F_{-1} \delta_i + v_i + o_p(1).$$

with v_i the vector containing individual v_{it}.

Testing for nonfundamentalness for a particular structural shock is achieved by performing an orthogonality test of the estimated structural shock with respect to the lags of the factors as proposed by Forni and Gambetti [2014]. Formally, an orthogonality test is performed with the following LM_T statistic :

$$LM_T = T \left(\frac{\widehat{\tilde{\eta}}_i' \mathbf{X_0} (\mathbf{X_0}' \mathbf{X_0})^{-1} \mathbf{X_0}' \widehat{\tilde{\eta}}_i}{\widehat{\tilde{\eta}}_i' \widehat{\tilde{\eta}}_i} \right)$$

[28] This holds for each element of the matrix $\tilde{A}_0$. The expression $o_p(1)$ means that this term converges in probability to zero

where $\mathbf{X_0}$ is the matrix containing the lagged values of the variable in the VAR (namely y_{t-1}) augmented by the lags of the factors f_{t-1}. The tests is carried out by regressing the estimated structural shock $\widehat{\tilde{\eta}_{it}}$ on y_{t-1} and f_{t-1} and the statistic is equal to TR^2 of the regression. The statistic is asymptotically distributed as a chi-squared distribution with a number of degree of freedom equals to the dimension of f_{t-1}. The test is equivalent to regressing the structural shocks $\tilde{\eta}_{it}$ on the lags of the factors orthogonal to y_{t-1}, namely $M_Y F_{-1}$.

The corresponding Wald statistic W_T for the orthogonality test is:

$$W_T = \frac{\hat{\delta}_i' \left(F_{-1}' M_Y F_{-1}\right) \hat{\delta}_i}{\hat{\sigma}_{vi}^2},$$

where $\hat{\delta}_i$ is the consistent estimator of δ_i and $\hat{\sigma}_{vi}^2$ is the estimator of the variance of v_{it}, *i.e.* the error term of the regression of $\hat{\tilde{\eta}}_{it}$ on $\hat{f}_{t-1}$. The R_i^2 associated to the linear regression of $\hat{\tilde{\eta}}_{it}$ on $\hat{f}_{t-1}$ is given by:

$$R_i^2 = \frac{\hat{\delta}_i' \left(F_{-1}' M_Y F_{-1}\right) \hat{\delta}_i}{\hat{\delta}_i' \left(F_{-1}' M_Y F_{-1}\right) \hat{\delta}_i + \hat{v}_i' \hat{v}_i}.$$

Using $\hat{v}_i' \hat{v}_i = T \times \hat{\sigma}_{vi}^2$, we obtain the desired result.

D Proof of Proposition 4:

The first part of proposition 4 is obtained by using that $V(\widehat{\tilde{\eta}}_{it}) = 1 = \hat{\delta}_i' \left(F_{-1}' M_Y F_{-1}\right) \hat{\delta}_i + \hat{\sigma}_{vi}^2$, so that variance of $\hat{\tilde{\eta}}_{it}$ can be rewritten as [29] $V(\widehat{\tilde{\eta}}_{it}) = R_i^2 + \hat{\sigma}_{vi}^2$. The estimator $\hat{\sigma}_{vi}^2 = (1 - R_i^2)$ is a consistent estimator of the expression $e_i' \left(\tilde{A}_0\right)^{-1} A_0 A_0' \left(\tilde{A}_0\right)^{-1\prime} e_i$ using the fact that $v_{it} = e_i' \tilde{A}_0^{-1} A_0 \eta_t$ and equation (C.2). Thus, R_i^2 is an estimator of the distance between the misspecified structural shock $\tilde{\eta}_i$ and the true structural shock $\eta_{it} = e_i' \eta_t$. When $R_i^2 = 0$ for all $i = 1, \ldots, n$, $\|\tilde{A}_0 - A_0\| = 0$.

To prove the second part of proposition 4, consider the true structural moving average representation of y_t :

$$y_t = A(L)\eta_t = A_i(L)\eta_{it} + A_{-i}(L)\eta_{-i,t}. \tag{D.1}$$

where $\eta_{-i,t}$ is the vector containing the true structural shock except η_{it}. By assumption, the vector $\eta_{-i,t}$ is orthogonal to all leads and lags of η_{it}. First suppose that $R_i^2 = 0$, the i-th structural shock is then correctly identified with the misspecified model. The corresponding structural moving average representation is then given by:

$$y_t = \tilde{A}(L)\tilde{\eta}_t = \tilde{A}_i(L)\eta_{it} + \tilde{A}_{-i}(L)\tilde{\eta}_{-i,t}. \tag{D.2}$$

since $\tilde{\eta}_{it} = \eta_{it}$ where $\tilde{A}_i(L)$ is the i-th column of $\tilde{A}(L)$ and $\tilde{A}_{-i}(L)$ is the matrix polynomial $\tilde{A}(L)$ after eliminating the its i-column. The vector $\tilde{\eta}_{-i,t}$ is the structural shocks vector containing the structural shocks other than the i-th structural shock identified with the misspecified model.[30] By

[29] The unit variance of $\widehat{\tilde{\eta}}_{it}$ is just the consequence of the normalization assumption of the structural shocks.

[30] Note that in general the vectors $\eta_{-i,t}$ and $\tilde{\eta}_{-i,t}$ are not of the same dimension because under the true model y_t is also function of the vector of reduced form error terms ϵ_{ft}.

construction $\tilde{\eta}_{-i,t}$ is also orthogonal to all leads and lags of the structural shock η_{it}. Representations (D.1) and (D.2) directly imply that $\tilde{A}_i(L) = A_i(L)$. Indeed, consider that this is not the case such as $\tilde{A}(L)\tilde{\eta}_{it} \neq A(L)\eta_{it}$, (D.1) and (D.2) implies that

$$y_t = A(L)\eta_t = \tilde{A}_i(L)\eta_{it} + \Big(A_i(L) - \tilde{A}_i(L)\Big)\,\eta_{it} + A_{-i}(L)\,\eta_{-i,t}. \tag{D.3}$$

which violates the fact that the component $\tilde{A}_i(L)\eta_{it}$ is orthogonal to all leads and lags of the $\tilde{\eta}_{-i,t}$ in (D.2). Consequently, the impulse response functions for the i-th identified structural shock from the misspecified model are equal to the true ones.

Now, by eq. (C.2), the R_i^2 is a continuous function of the vector parameters $\beta_{\mathbf{f}} = vec(B_f)$. By the data generating process, true impulse response functions are continuous respective to the parameters model and in particular continuous in respect to $\beta_{\mathbf{f}}$. We can easily show that the impulse response functions $\tilde{A}(L)$ depends continuously on the true second moments of Y_T which are continuous function of the well-specified model parameters. As consequence, the bias of the individual impulse responses for the structural shock i for each variable k and lag l, namely, $bias_{i,k,l} = |\tilde{A}_{i,k,l} - A_{i,k,l}|$ is a continuous function of $\beta_{\mathbf{f}}$. By continuity, when R_i^2 is small, the $bias_{i,k,l}$ is small for any variable and lag.

To prove the third part of proposition 4, consider the variance of a variable j attributable to structural shock η_{it}. On impact, it is given by: $e_j'\tilde{A}_0 e_i Var(\tilde{\eta}_{it}) e_i'\tilde{A}_0' e_j' = e_j'\tilde{A}_0 e_i R_i^2 e_i'\tilde{A}_0' e_j' + e_j'\tilde{A}_0 e_i (1 - R_i^2) e_i'\tilde{A}_0' e_j' + o_p(1)$ with $(1 - R^2)$ a consistent estimator of $e_i'\left(\tilde{A}_0\right)^{-1} A_0 A_0' \left(\tilde{A}_0\right)^{-1'} e_i$ as aforementioned. The R_i^2 is then a consistent empirical measure of the discrepancy between the misspecified variance on the impact attributable to a particular shock and its true one.

E Proof of Proposition 5 :

Applying the previous computations to this simple two–variable example yields the following expression for the first structural shock : $\tilde{\eta}_{1t} = \delta_1'\hat{f}_{t-1} + e_1'\tilde{A}_0^{-1}A_0\eta_t + o_p(1) = \delta_1'\hat{f}_{t-1} + \frac{a_{0,11}}{\tilde{a}_{0,11}}\eta_{1t} + o_p(1)$ and $e_1'\left(\tilde{A}_0\right)^{-1} A_0 A_0' \left(\tilde{A}_0\right)^{-1'} e_1 = \left(\frac{a_{0,11}}{\tilde{a}_{0,11}}\right)^2$. Consequently, $(1 - R_1^2)$ is a consistent estimator of this term. Hence, $V\left(\widehat{\tilde{\eta}}_{1t}\right) = \hat{\delta}_1'\left(F_{-1}'M_Y F_{-1}\right)\hat{\delta}_1 + \widehat{\left(\frac{a_{0,11}}{\tilde{a}_{0,11}}\right)^2} \equiv R_1^2 + (1 - R_1^2)$. For R_1^2 small, a first order expansion this implies that $\widehat{\tilde{a}}_{0,11} \simeq \left(1 + \frac{1}{2}R_1^2\right) a_{0,11}$. So, the relative bias is $\frac{\widehat{\tilde{a}}_{0,11} - a_{0,11}}{a_{0,11}} \simeq \frac{1}{2}R_1^2$. Consider now the second structural shock η_{2t}. Again, using our calculations above yields $\tilde{\eta}_{2t} = \delta_2'\hat{f}_{t-1} + e_2'\tilde{A}_0^{-1}A_0\eta_t + o_p(1) = \delta_2'\hat{f}_{t-1} + \left[\frac{a_{0,21}}{\tilde{a}_{0,22}} - \frac{a_{0,11}}{\tilde{a}_{0,11}}\frac{\tilde{a}_{0,21}}{\tilde{a}_{0,22}}\right]\eta_{1t} + \frac{a_{0,22}}{\tilde{a}_{0,22}}\eta_{2t} + o_p(1)$. The expression for $\tilde{\eta}_{2t}$ is a function of the relative bias for the three terms in the matrix A_0. This implies the following variance of the second structural shock $V\left(\widehat{\tilde{\eta}}_{2t}\right) = \hat{\delta}_2'\left(F_{-1}'M_Y F_{-1}\right)\hat{\delta}_2 + \widehat{\Theta}^2 + \widehat{\left(\frac{a_{0,22}}{\tilde{a}_{0,22}}\right)^2} \equiv R_2^2 + (1 - R_2^2)$ where $\Theta = \left[\frac{a_{0,21}}{\tilde{a}_{0,22}} - \frac{a_{0,11}}{\tilde{a}_{0,11}}\frac{\tilde{a}_{0,21}}{\tilde{a}_{0,22}}\right]$. This implies that $\widehat{\Theta}^2 + \widehat{\left(\frac{a_{0,22}}{\tilde{a}_{0,22}}\right)^2} = (1 - R_2^2)$ Consequently, $\widehat{\tilde{a}}_{0,22} \leq (1 - R_2^2)^{-1/2} a_{0,22} \leq \left(1 + \frac{1}{2}R_2^2\right) a_{0,22} + o\left(R_2^2\right) a_{0,22}$ and $\widehat{\Theta} \leq (1 - R_2^2)^{1/2}$.

F Data

- Hours: BLS, Series Id: PRS85006033, Nonfarm Business sector, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Consumption: BEA, Table 1.1.3. Real Gross Domestic Product, Quantity Indexes, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- Investment: BEA, Table 1.1.3. Real Gross Domestic Product, Quantity Indexes, 1947Q1-2012Q3, seasonally adjusted, downloaded: 12/2012
- TFP: Utilization-adjusted quarterly-TFP series for the U.S. Business Sector, produced by John Fernald, series ID: dtfp_util, 1947Q1-2012Q3, downloaded: 12/2012
- Stock Prices: S&P500 index deflated by CPI, obtained from the homepage of Robert J. Shiller.