

Using implied probabilities to improve the estimation of  
unconditional moment restrictions for weakly dependent data<sup>1</sup>

Alain Guay

*University of Quebec at Montreal (UQAM)*

*CIRPÉE, CIREQ and L.E.A.D.*

Florian Pelgrin

*EDHEC Business School, Lille*

*and*

*Faculty of Business and Economics (HEC)*

*University of Lausanne*

This version: August 2013

<sup>1</sup>Florian Pelgrin gratefully acknowledges financial support from the National Center of Competence in Research “Financial Valuation and Risk Management”. The National Centers of Competence in Research (NCCR) are a research instrument of the Swiss National Science Foundation. Alain Guay gratefully acknowledges financial support from le Fonds québécois de la recherche sur la société et la culture (FQRSC). All proofs and extensions are available on the following link, <http://www.er.uqam.ca/nobel/r27460/>. Email addresses : Florian Pelgrin, [florian.pelgrin@edhec.edu](mailto:florian.pelgrin@edhec.edu) and Alain Guay: [guay.alain@uqam.ca](mailto:guay.alain@uqam.ca)

## **Abstract**

In this paper, we investigate the use of implied probabilities (Back and Brown, 1993) to improve estimation in unconditional moment conditions models. Using the seminal contributions of Bonnal and Renault (2001) and Antoine, Bonnal and Renault (2007), we propose two three-step Euclidian empirical likelihood (3S-EEL) estimators for weakly dependent data. Both estimators make use of a control variates principle that can be interpreted in terms of implied probabilities in order to achieve higher-order improvements relative to the traditional two-step GMM estimator. A Monte Carlo study reveals that the finite and large sample properties of the three-step estimators compare favorably to the existing approaches: the two-step GMM and the continuous updating estimator.

*JEL classification:* C13, C14, E31

*Keywords:* Information-based inference, Implied probabilities, Generalized method of moments, linear rational expectation models.

# 1 Introduction

A number of studies have recently revealed that the efficient Generalized Method of Moments (GMM) estimator introduced by Hansen (1982) may have a large bias for sample sizes typically encountered in applied economics.<sup>1</sup> Alternative estimators based on a one-step procedure that are first-order equivalent and achieve higher-order improvements (Newey and Smith, 2004; Anatolyev, 2005) relative to the two-step GMM estimator have been suggested to address this problem. Newey and Smith (2004) have shown that these alternative estimators share a common structure, being members of a class of generalized empirical likelihood (GEL) estimators. These alternative estimators include the Continuous Updating Estimator (CUE) proposed by Hansen, Heaton and Yaron (1996), the Empirical Likelihood (EL) estimator of Qin and Lawless (1994), the Exponential Tilting (ET) estimator of Kitamura and Stutzer (1997) and Imbens, Spady and Johnson (1998), and the exponentially tilted empirical likelihood estimator of Schennach (2007).

On the other hand, Antoine, Bonnal, and Renault (2007) propose, in an i.i.d context, a three-step Euclidian empirical likelihood (3S-EEL) estimator based on a Chi-square distance where the last step consists of solving the first order conditions (FOC) of the Euclidian Empirical Likelihood (EEL) estimator given some efficient estimators of the Jacobian and the optimal weighting matrices evaluated at an efficient second-step estimator (e.g., the 2S-GMM estimator). As explained in Bonnal and Renault (2001), and Antoine, Bonnal and Renault (2007), efficiency results from the fact that the (Euclidean) implied probabilities (Back and Brown, 1993; Brown and Newey, 1998), which assign a weight to each observation in the sample such that the sample moment conditions are satisfied, provide population expectation estimates by using the overidentifying moment conditions as control variates. Importantly the 3S-EEL estimator has at least two appealing properties. First, it is higher-order equivalent to the empirical likelihood estimator—their difference being  $\mathcal{O}_p(T^{-3/2})$ .<sup>2</sup> Second, the 3S-EEL estimator is more computationally convenient than the one-step alternatives : the first two steps involve quadratic optimization and the last step amounts of solving a GMM-like first order condition—this

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<sup>1</sup>For instance, see the special issue of *Journal of Business and Economic Statistics*, July, 1996.

<sup>2</sup>Throughout of our paper, we assume that there is absence of misspecification. Dovonon (2010) studies the 3S-EEL estimator under model misspecification.

sharply contrasts with the implementation of a nested optimization algorithm for the class of GEL estimators (Kitamura, 2006).<sup>3</sup> The numerical implementation of the 3S-EEL estimator and the simple interpretation of quadratic optimizations are critical when comparing the alternatives to the 2S-GMM estimator—their finite sample properties being a second key issue in these comparisons.

In this respect, we reconsider the use of the (smoothed) Euclidean implied probabilities to improve estimation in unconditional moment conditions models with weakly dependent data. Notably we propose two extensions of Antoine, Bonnal, and Renault (2007) for time series data. The first estimator leads to the same GMM-like first-order conditions in the third step after considering the smoothed moment conditions and the smoothed Euclidean implied probabilities. The second smoothed 3S-EEL estimator (denoted 3SW-EEL) rewrites the third step meaning that the estimator of interest intervenes in both the sample average of the smoothed moment conditions and the reweighted smoothed derivative estimator of the Jacobian matrix. Both estimators can be interpreted using the long term control variates principle of Bonnal and Renault (2001). Importantly, both estimators achieve a higher-order equivalence to the smoothed empirical likelihood (SEL) estimator (up to an order  $\mathcal{O}_p((2K_T + 1)/T^{3/2})$  where  $K_T$  is the smoothing parameter of the uniform truncated kernel of the smoothed moment conditions) and are more computationally convenient than the smoothed GEL estimators.

Obviously these two estimators are closely related to the contributions of Back and Brown (1993), Bonnal and Renault (2001), Anatolyev (2005), Antoine, Bonnal and Renault (2007), and Smith (2011). On the one hand, Back and Brown (1993) settle the implied probabilities in a time series context. On the other hand, Bonnal and Renault (2001) provide the interpretation regarding long term control variables and make explicit the relationship with HAC estimation for the CUE. Antoine, Bonnal and Renault (2007) reconsider the arguments of Newey and Smith (2004) in the context of Euclidean Empirical Likelihood and detail (among others) the control variates principle (and the "efficient use of the information content of estimating equations") as well as the shrinkage procedure for the implied probabilities. Finally the higher-order efficiency is extensively studied by Anatolyev (2005) in terms

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<sup>3</sup>Fan, Gentry and Li (2011) also provide a new class of estimators that are less computationally demanding and share the same higher order properties as GEL estimators to any given order provided some conditions in an i.i.d. context.

of weakly dependent data whereas Smith (2011) defines GEL methods using weakly dependent data through the smoothing of moment conditions. In so doing, our proposed estimators combine these arguments.

At the same time, a key contribution of this paper is also to study extensively the finite and large samples properties of the proposed estimators relative to the 2S-GMM estimator and the CUE. Indeed, the finite (large) sample properties of the 3S-EEL have not been thoroughly studied in an i.i.d. context with the exceptions of Dovonon (2010) and Fan, Gentry and Li (2011) while no comparative studies, to the best of our knowledge, have been carried out for weakly dependent data. In addition, while Monte Carlo studies on one-step estimators are almost in an i.i.d. context and tend to be about some small-scale *ad hoc* models, we use a more realistic model. To this end, we assume that the data generating process is given by the reduced-form of a univariate linear rational expectations model. This class of models is often used in applied macroeconomics, as for instance any log-linearized Euler equation in a dynamic stochastic general equilibrium model. Therefore, our results are of particular interest and may provide some useful guidelines in applied economics. Our simulation results provide evidence that the proposed estimators are competitive relative to the 2S-GMM estimator and the CUE. Indeed, they generally perform better in terms of median (mean) bias and RMSE than the 2S-GMM estimator. Among the proposed smoothed three-step estimators, the 3SW-EEL estimator has generally better finite and large sample median (mean) bias properties than the time-series extension of the 3S-EEL estimator. On the other hand, the smoothed 3S-EEL estimator performs very well in terms of RMSE, especially when the number of instruments is small. At the same time, the smoothed 3SW-EEL estimator involves a higher but tractable (with respect to the CUE) computation time than the time-series extension of the 3S-EEL estimator.

The rest of the paper is organized as follows. In Section 2, we present the 3S-EEL with i.i.d. observations. Section 3 presents the two smoothed three-step estimators. In Section 4, we provide Monte Carlo simulations. The last section concludes. All proofs are relegated to the Appendix.

## 2 The three-step Euclidean empirical likelihood estimator with i.i.d. observations

In this section, we first present the three-step Euclidean likelihood estimator (Bonnal and Renault, 2001; Antoine, Bonnal, and Renault, 2007) in an i.i.d. context.

We consider models specified by a finite number of moment conditions. More precisely, let  $\{z_t : t = 1, \dots, T\}$  be  $\mathbb{R}^l$ -valued i.i.d. data, where  $T$  denotes the sample size. Let  $g(z_t, \theta) : H \times \Theta \rightarrow \mathbb{R}^q$ , where  $H \subset \mathbb{R}^l$  and  $\Theta \subset \mathbb{R}^p$ , and  $\theta \in \Theta$  denote respectively the  $p$ -vector of unknown parameters and the parameter space. The number of moment conditions,  $q$ , is greater than or equal to the number of parameters,  $p$ . The true parameter vector  $\theta^0$  satisfies the unconditional moment conditions:

$$E[g(z_t, \theta^0)] = 0 \tag{1}$$

where  $E[\cdot]$  denotes the expectation operator with respect to the unknown distribution of  $z_t$ .<sup>4</sup>

As proposed by Bonnal and Renault (2001), and Antoine, Bonnal, and Renault (2007), the three-step Euclidean likelihood estimator first involves two quadratic optimization problems in order to determine an efficient GMM estimator (e.g., the two-step efficient GMM estimator) and the (Euclidean) implied probabilities (Back and Brown, 1993),  $\{\pi_t, t = 1, \dots, T\}$ . Then a third step solves a GMM-like first-order condition in which the Jacobian and the variance-covariance matrix of the moment conditions are estimated using the Euclidean implied probabilities as weights in population

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<sup>4</sup>For economy of notation,  $g_t(\theta)$  is often used to denote  $g(z_t; \theta)$  in the sequel.

expectation estimates. Consider the following notation:

$$G_t(\theta) = \frac{\partial g_t(\theta)}{\partial \theta'}, \quad V_T(\theta) = T^{-1} \sum_{t=1}^T g_t(\theta) (g_t(\theta) - \bar{g}_T(\theta))', \quad \bar{g}_T(\theta) = \frac{1}{T} \sum_{t=1}^T g_t(\theta)$$

$$\pi_t(\theta) = \frac{1}{T} - \frac{1}{T} (g_t(\theta) - \bar{g}_T(\theta))' V_T^{-1} \bar{g}_T(\theta) \quad (2)$$

$$\tilde{G}_T(\theta) = \sum_{t=1}^T \pi_t(\theta) G_t(\theta)' \quad (3)$$

$$\tilde{\Omega}_T(\theta) = \sum_{t=1}^T \pi_t(\theta) g_t(\theta) g_t(\theta)'. \quad (4)$$

The 3S-EEL estimator,  $\hat{\theta}_T^{3S}$ , is then defined to be the solution of the following p equations:

$$\left[ \tilde{G}_T(\hat{\theta}_T) \right]' \left[ \tilde{\Omega}_T(\hat{\theta}_T) \right]^{-1} \bar{g}_T(\hat{\theta}_T^{3S}) = 0 \quad (5)$$

where  $\hat{\theta}_T$  is some efficient GMM estimator of  $\theta$  (e.g., the 2S-GMM estimator). The closed-form (Euclidean) implied probabilities  $\{\pi_t, t = 1, \dots, T\}$  are the solution of a (constrained) quadratic optimization problem (Bonnal and Renault, 2001; Antoine, Bonnal and Renault, 2007)—the minimized objective function is referred to as the EEL or the Chi-square (Owen, 2001) and it belongs to the family of power-divergence statistics introduced by Cressie and Read (1984) or the class of minimum discrepancy estimators (Corcoran, 1998).<sup>5</sup> Those implied probabilities are the empirical measure counterparts to the expectation operator in Eq. (1), which ensure that the moment conditions hold true in the sample, and they differ in general with the empirical measure  $\{\pi_t = T^{-1}, t = 1, \dots, T\}$ , which is obtained from the maximization of the nonparametric log-likelihood subject to the constraints  $0 < \pi_t < 1$  ( $t = 1, \dots, T$ ) and  $\sum_{t=1}^T \pi_t = 1$ .

Two points are worth commenting. First, following Bonnal and Renault (2001), and Antoine, Bonnal and Renault (2007), the efficient Jacobian and the variance-covariance matrix of the moment

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<sup>5</sup>For further details, see Antoine, Bonnal, and Renault (2007, p. 465). Notably Antoine, Bonnal, and Renault (2007, Theorem 3.3.) show that the implied probabilities of Back and Brown (1993) can be revisited in a CUE context when working with an augmented set of moment conditions.

conditions in the third step can be written as:

$$\tilde{G}_T(\hat{\theta}_T) = G_T(\hat{\theta}_T) - Cov_T \left[ G_t(\hat{\theta}_T), g_t(\hat{\theta}_T) \right] V_T(\hat{\theta}_T)^{-1} \bar{g}_T(\hat{\theta}_T) \quad (6)$$

and

$$\tilde{\Omega}_T(\hat{\theta}_T) = V_T(\hat{\theta}_T) - Cov_T \left[ g_t(\hat{\theta}_T)g_t(\hat{\theta}_T)', g_t(\hat{\theta}_T) \right] V_T(\hat{\theta}_T)^{-1} \bar{g}_T(\hat{\theta}_T) \quad (7)$$

where  $G_T(\theta) = \frac{1}{T} \sum_{t=1}^T G_t(\theta)'$  and  $Cov_T[h_{1t}(\theta), h_{2t}(\theta)] = T^{-1} \sum_{t=1}^T [h_{1t}(\theta) - \bar{h}_{1T}(\theta)] [h_{2t}(\theta) - \bar{h}_{2T}(\theta)]'$  for any  $\mathbb{R}^\ell$ -valued functions  $h_1$  and  $h_2$ . This highlights the interpretation in terms of the control variates principle (see Antoine, Bonnal, and Renault, 2007, p. 466)—an unbiased estimator of  $E(h(Z))$  for any function  $h$  can be determined by considering  $\bar{h}_T - a' \bar{g}_T(\theta)$  with  $a = Cov[h(\theta)h(\theta)', g(\theta)] V(\theta)^{-1}$ . The variance reduction resulting from the control variable principle is asymptotically semiparametrically efficient with respect to the moment conditions in Eq. (1).

Second, the generalized empirical likelihood estimator also solves a GMM-like first-order condition as in Eq. (5). The two main differences are that, in the case of the 3S-EEL estimator, (i) the (Euclidean) implied probabilities have a closed-form solution and (ii) the Jacobian and the variance-covariance matrix of the moment conditions are evaluated at an efficient GMM estimator (e.g., the 2S-GMM estimator). Consequently the 3S-EEL estimator is less demanding from a numerical point of view and thus more computationally convenient than higher-order equivalent GEL estimators. At the same time, the 3S-EEL estimator may suffer from computational inefficiency due to some non-positive Euclidean implied probabilities. However, as shown by Antoine, Bonnal and Renault (2007, Theorem 2.2.), the use of a shrinkage correction avoids such non-positive implied probabilities while preserving the asymptotic equivalence (at least at the first-order) between the corrected 3S-EEL (using the shrunk implied probabilities) and the non-corrected 3S-EEL estimator.

### 3 Smoothed three-step Euclidean likelihood estimators

In this section, we reconsider the 3S-EEL estimator of Antoine, Bonnal and Renault (2007) in unconditional moment conditions models with weakly dependent data. In the sequel, the moment conditions

are still defined by Eq. (1) but  $\{z_t : t = 1, \dots, T\}$  are  $\mathbb{R}^l$ -valued time series data. Moreover we mainly impose the same assumptions as in Anatolyev (2005) (see Assumptions A in Appendix 1).<sup>6</sup>

In order to compare the higher asymptotic properties of the two extensions of the 3S-EEL with the smoothed empirical likelihood estimator (SEL) in the case of weakly dependent data, we first need to provide the definition of this latter estimator. In so doing, moment conditions have to be smoothed with an appropriate kernel to account for the presence of temporal dependence. According to Bonnal and Renault (2001) and Smith (2011), the smoothed moment conditions are defined by:

$$g_{tT}(\theta) = \frac{1}{S_T} \sum_{s=t-T}^{t-1} k\left(\frac{s}{S_T}\right) g(z_{t-s}, \theta) \quad (8)$$

where  $t = 1, \dots, T$ ,  $S_T$  is a bandwidth parameter, and  $k(\cdot)$  is a kernel function with  $k_j = \int_{-\infty}^{\infty} k(a)^j da$ . Some sufficient regularity conditions on the bandwidth parameter  $S_T$  and the kernel function  $k(\cdot)$  (Smith, 2011) must be imposed for consistency results, and especially for the consistency of the long-run variance-covariance matrix of the moment conditions. In particular these conditions ensure that (i)  $S_T$  has similar conditions to those in Andrews (1991, Theorem 1) and (ii) the induced kernel is a member of the positive semi-definite class of kernels used in HAC covariance matrix estimation.

Following Anatolyev (2005) and Smith (2011), the smoothed empirical likelihood estimator and the corresponding implied probabilities can be defined as follows. Let  $\rho$  denote a function of a scalar  $\phi$  that is concave on its domain—an open interval  $\Phi$  that contains zero. Let  $\hat{\Lambda}_T(\theta) = \{\lambda : k\lambda'g_{tT}(\theta) \in \Phi, t = 1, \dots, T\}$  and  $k = \frac{k_1}{k_2}$ . Then, under suitable regularity conditions, the smoothed empirical likelihood (SEL) estimator is a solution to the saddle point problem:

$$\hat{\theta}_T^{SEL} = \arg \min_{\theta \in \Theta} \sup_{\lambda \in \hat{\Lambda}_T(\theta)} \frac{1}{T} \sum_{t=1}^T [\rho(k\lambda'g_{tT}(\theta)) - \rho_0] \quad (9)$$

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<sup>6</sup>The main exception is that we make use of the uniform kernel of Kitamura and Stutzer (1997). It is also worth noting that the results presented below, especially those of Proposition 1, can be derived under weaker assumptions than Assumptions A in Appendix 1. However, according to Anatolyev (2005), Assumptions A turn out to be quite convenient for higher-order asymptotics.

where  $\rho(\phi) = \ln(1 - \phi)$ . Accordingly, the smoothed empirical likelihood implied probabilities are given by:

$$\pi_t^{SEL}(\hat{\theta}_T^{SEL}) = \frac{1}{\left(T \left(1 - \hat{\lambda}'_T g_{tT}(\hat{\theta}_T^{SEL})\right)\right)} \quad (10)$$

where  $\hat{\lambda}_T = \arg \max_{\lambda \in \hat{\Lambda}(\hat{\theta}_T)} T^{-1} \sum_{t=1}^T \ln \left(1 - \lambda' g_{tT}(\hat{\theta}_T^{SEL})\right)$ .

We can now proceed with our proposed extensions. Taking the 3S-EEL estimator of Antoine, Bonnal, and Renault (2007), we need to redefine Eq. (2), (3), (4) and (5) in a time-series context. First, note that Eq. (8) implies that the smoothed derivatives of the moment conditions are given by:

$$G_{tT}(\theta) = \frac{1}{S_T} \sum_{s=t-T}^{t-1} k\left(\frac{s}{S_T}\right) \frac{\partial g}{\partial \theta'}(z_{t-s}, \theta).$$

As proposed by Kitamura and Stutzer (1997), the uniform truncated kernel is used for  $k(\cdot)$ . On the one hand, Bonnal and Renault (2001) make explicit the reason why Kitamura and Stutzer (1997) consider the uniform truncated kernel.<sup>7</sup> On the other hand, Smith (2005, 2011) discusses examples of appropriate kernels and bandwidth parameters which ensure that GEL estimators are first-order asymptotically equivalent to efficient GMM estimators. Among them, the uniform kernel proposed by Kitamura and Stutzer (1997) induces the Bartlett kernel for the estimation of the long-run variance-covariance matrix of the moment conditions. Finally, Anatolyev (2005) extends the generalized empirical likelihood estimator of Newey and Smith (2004) to a time series context and shows that, among positive kernels, only the uniform truncated kernel removes the bias component (at order  $T^{-1}$ ) involved by the third moments of the moment conditions.<sup>8</sup>

Following Kitamura and Stutzer (1997), the uniform truncated kernel yields the smoothed moment

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<sup>7</sup>Moreover, Bonnal and Renault (2001) provide the relationship between the long-run control variates principle and the HAC estimation for the CUE.

<sup>8</sup>If the third moments of the moment conditions are zeroes, the control variates principle does not permit to improve the estimation.

conditions:

$$g_{tT}(\theta) = \frac{1}{2K_T + 1} \sum_{s=-K_T}^{K_T} g(z_{t-s}, \theta) \quad (11)$$

where  $K_T$  satisfies the conditions  $K_T \rightarrow \infty$ , and  $K_T/T^2 \rightarrow 0$  as  $T \rightarrow \infty$ . Hence the optimal bandwidth parameter rate is  $K_T = \mathcal{O}(T^{1/3})$ . According to Bonnal and Renault (2001), it follows that a consistent estimator of the centered long-run variance-covariance matrix of the moment conditions  $V_T$  (evaluated at a given  $\theta$ ) is given by:

$$\hat{V}_T(\theta) = \frac{1}{T}(2K_T + 1) \sum_{t=1}^T [g_{tT}(\theta) - \bar{g}_T(\theta)] g_{tT}(\theta)'$$

and a consistent estimator of the uncentered long-run variance-covariance matrix of the moment conditions  $\Omega_T$  (evaluated at a given  $\theta$ ) is defined to be:

$$\hat{\Omega}_T(\theta) = \frac{1}{T}(2K_T + 1) \sum_{t=1}^T g_{tT}(\theta) g_{tT}(\theta)'.$$

Using the smoothed moment conditions, the smoothed Euclidean implied probabilities can be computed using either the results of Bonnal and Renault (2001) and Antoine, Bonnal, and Renault (2007) or the duality between the minimum distance estimators based on the Cressie-Read family of discrepancies and the generalized empirical likelihood estimators (Newey and Smith, 2004; Smith, 2011).<sup>9</sup> Following Bonnal and Renault (2001), the closed-form expression of the SEEL implied probabilities evaluated at  $\theta$  can be written with the centered (respectively, the uncentered) estimator of the long-run covariance matrix:

$$\pi_t^{SEEL}(\theta) = \frac{1}{T} - \frac{1}{T}(2K_T + 1) [g_{tT}(\theta) - \bar{g}_T(\theta)]' \hat{V}_T(\theta)^{-1} \bar{g}_T(\theta) \quad (12)$$

or

$$\pi_t^{SEEL}(\theta) = \frac{1}{T} - \frac{1}{T}(2K_T + 1) g_{tT}(\theta)' \hat{\Omega}_T(\theta)^{-1} \bar{g}_T(\theta). \quad (13)$$

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<sup>9</sup>The proof of Proposition 1 follows the second approach.

In this respect, Proposition 1 provides the difference regarding the order in probability between the smoothed Euclidean implied probabilities and the ones of the smoothed empirical likelihood (SEL) estimator. The latter result is crucial to derive the asymptotic higher-order difference between the smoothed three-step EEL estimators and the SEL estimator.

**Proposition 1** *Suppose that Assumptions A hold true, for any efficient first-order equivalent estimator  $\hat{\theta}_T$  of  $\theta$ ,*

$$\pi_t^{SEL}(\hat{\theta}_T) = \pi_t^{SEEL}(\hat{\theta}_T) + \mathcal{O}_p((2K_T + 1)/T^2)$$

*uniformly over  $t = 1, \dots, T$ .*

**Proof:** see Appendix 1.

Taking Proposition 1, we are now in a position to present estimators with weakly dependent data using the results of Bonnal and Renault (2001) and Antoine, Bonnal and Renault (2007). The first smoothed three-step estimator is the one proposed by Antoine et al. (2007) but for weakly dependent data. As stated in Definition 1, this estimator solves the smoothed version of the  $p$  first-order conditions (Eq. 5) after evaluating the Jacobian and the weighting matrices via a control variates principle in the same spirit as in Eq. (6) and (7)—the long run control variates approach of Bonnal and Renault (2001).

**Definition 1** *The smoothed 3S-EEL estimator,  $\hat{\theta}_T^{S3S}$ , is the solution of the following  $p$  equations:*

$$\left[ \sum_{t=1}^T \pi_t^{SEEL}(\hat{\theta}_T) G_{tT}(\hat{\theta}_T) \right]' \left[ (2K_T + 1) \sum_{t=1}^T \pi_t^{SEEL}(\hat{\theta}_T) g_{tT}(\hat{\theta}_T) g_{tT}(\hat{\theta}_T)' \right]^{-1} \frac{1}{T} \sum_{t=1}^T g_{tT}(\hat{\theta}_T^{S3S}) = 0 \quad (14)$$

*where  $\hat{\theta}_T$  is an efficient estimator of  $\theta$  and  $\pi_t^{SEEL}(\cdot)$  is defined in Eq. (12) or Eq. (13).*

The smoothed 3S-EEL estimator,  $\hat{\theta}_T^{S3S}$ , makes use of a reweighted smoothed derivative estimator of the Jacobian and of a reweighted smoothed estimator of the variance-covariance matrix of the moment conditions—both being evaluated at an efficient estimator of  $\theta$  (e.g., the 2S-GMM estimator). Following Bonnal and Renault (2001) and Smith (2011), these reweighted smoothed estimators efficiently

incorporate the moment information of Eq. (1) for weakly dependent data. Taking the closed-form solution of the implied probabilities, the computation of the smoothed 3S-EEL estimator is much easier than that of other smoothed GEL estimators whether the smoothed moment conditions,  $\sum_{t=1}^T g_{tT}(\hat{\theta}_T^{SS})$ , are either linear or nonlinear.

A second estimator, denoted  $\hat{\theta}_T^{S3SW}$  (W for weighting matrix), consists in solving the  $p$  equations as in Definition 1 with the main difference that the estimator of interest intervenes now in both the sample average of the smoothed moment conditions and the reweighted smoothed derivative estimator of the Jacobian. This alternative estimator is more in the spirit of the standard GMM estimator in the sense that only the weighting matrix is evaluated at the estimator  $\hat{\theta}_T$ .

**Definition 2** *The smoothed 3SW-EEL estimator,  $\hat{\theta}_T^{S3SW}$ , is the solution of the following  $p$  equations:*

$$\left[ \sum_{t=1}^T \pi_t^{SEEL}(\hat{\theta}_T^{S3SW}) G_{tT}(\hat{\theta}_T^{S3SW}) \right]' \left[ (2K_T + 1) \sum_{t=1}^T \pi_t^{SEEL}(\hat{\theta}_T) g_{tT}(\hat{\theta}_T) g_{tT}(\hat{\theta}_T)' \right]^{-1} \frac{1}{T} \sum_{t=1}^T g_{tT}(\hat{\theta}_T^{S3SW}) = 0$$

where  $\hat{\theta}_T$  is an efficient estimator of  $\theta$  and  $\pi_t^{SEEL}(\cdot)$  is defined in Eq. (12) or Eq. (13).

As a result, the smoothed 3SW-EEL estimator is computationally more demanding than the smoothed 3S-EEL. This might lead to an issue regarding the control of the computational burden of this estimator relative to the smoothed 3S-EEL estimator—this point is further discussed in the next section. At the same time, the smoothed 3SW-EEL estimator remains more computationally convenient than the smoothed CUE or the SEL estimator.

We now discuss the asymptotic properties of the estimator presented in Definitions 1 and 2. Both estimators are asymptotically higher-order equivalent to the SEL estimator up to order  $\mathcal{O}_p((2K_T + 1)/T^{3/2})$ . Indeed, starting from Anatolyev (2005), we show that the second-order asymptotic bias of the proposed estimators lacks some bias components with respect to the 2S-GMM estimator. More specifically, both smoothed three-step estimators remove (i) the bias component resulting from the correlation between the moment conditions and their derivatives and (ii) the bias component associated with the third moments by using an appropriate choice of the kernel. Finally, even with moment conditions serially

uncorrelated but not i.i.d. across time, Anatolyev (2005) states that the SEL estimator tends to reduce the bias—a property shared by both smoothed three-step EEL estimators.

The next proposition sets forth the higher-order equivalence between the SEL estimator and the smoothed 3SW-EEL estimator.

**Proposition 2** *Under Assumptions A in Appendix 1, the smoothed 3SW-EEL estimator,  $\hat{\theta}_T^{S3SW}$ , satisfies*

$$\hat{\theta}_T^{S3SW} - \hat{\theta}_T^{SEL} = \mathcal{O}_p\left((2K_T + 1)/T^{3/2}\right)$$

*and thus achieves the same asymptotic bias (up to  $\mathcal{O}_p(T^{-1})$ ) as the SEL estimator.*

**Proof:** see Appendix 1.

Proposition 2 also holds for the time series extension of the 3S-EEL. The characterization of the asymptotic higher-order properties of the smoothed three-step estimators in Proposition 2 leads to several remarks. First, the result depends on the smoothing parameter  $K_T$  used to implement the uniform truncated kernel. This bandwidth parameter satisfies sufficient regularity conditions for the consistency results. From a practical view, the smoothing parameter  $K_T$  is chosen in the simulation experiments according to the data-dependent procedure proposed by Newey and West (1994). More specifically,  $K_T$  is set to the integer value of  $(m_T - 1)/2$  where  $m_T$  is the lag length chosen by the data-driven procedure of Newey and West (1994). Second, for i.i.d. data,  $K_T$  is fixed to zero and we retrieve the result of Antoine et al. (2007) that the asymptotic higher-order difference is  $\mathcal{O}_p(T^{-3/2})$ . Third, the smoothed 3S-EEL, 3SW-EEL and the SEL estimators have the same bias-order, namely  $\mathcal{O}(T^{-1})$ , so that the higher-order asymptotic derivations in Anatolyev (2005) can be used in order to determine a bias-corrected version of both estimators.<sup>10</sup>

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<sup>10</sup>See Appendix 2.

## 4 Simulation Experiments

In this section, we examine the finite sample properties of the CUE, the 2S-GMM estimator and the smoothed 3S-EEL and 3SW-EEL estimators.

### 4.1 The data generating process

We assume that the data generating process (DGP) is based on the (hybrid) quasi-structural form of a univariate rational expectations model, as for instance any log-linearized Euler equation in a dynamic stochastic general equilibrium model. Following Mavroiedis (2004), and Nason and Smith (2008), the forcing variable is driven by an autoregressive process of order 2. The dynamic specification is thus given by:

$$\begin{aligned}y_t &= \gamma_f E_t y_{t+1} + \gamma_b y_{t-1} + \lambda x_t + \epsilon_t \\x_t &= \rho_1 x_{t-1} + \rho_2 x_{t-2} + v_t\end{aligned}$$

where  $\rho_1$  and  $\rho_2$  satisfy the standard weak stationarity conditions,  $\gamma_f$ ,  $\gamma_b$  and  $\lambda$  are generally nonlinear functions of some structural parameters, say  $\theta \in \Theta$ ,  $\epsilon_t$  is an exogenous shock with zero mean and variance  $\sigma_\epsilon^2$ , and  $v_t$  is the innovation process. The variance-covariance matrix of the error terms is defined by:

$$\Sigma = \begin{pmatrix} \sigma_\epsilon^2 & \rho \sigma_\epsilon \sigma_v \\ \rho \sigma_\epsilon \sigma_v & \sigma_v^2 \end{pmatrix}$$

where  $\rho$  is the correlation coefficient.

The estimation methods use the sample version of the following moment conditions:

$$E [Z_t (y_t - \lambda x_t - \gamma_f y_{t+1} - \gamma_b y_{t-1})] = 0 \tag{15}$$

where the vector  $Z_t$  denotes the set of instruments.

As well-explained by Nason and Smith (2008), identification requires the predictability of the future forcing variable values beyond that provided by the current ones, or current or lagged endogenous variable. Since the forcing variable,  $x_t$ , follows a second-order autoregressive process, it is a necessary condition for identification. However, even though this necessary condition is respected, the strength of identification should be taken into consideration more precisely (Kleibergen and Mavroiedis, 2009). To this end, we follow the approach recommended by Mavroiedis (2004, 2005), i.e. we report the concentration parameter of the reduced-form model.<sup>11</sup>

## 4.2 Finite and large sample properties of the estimators

We report Monte Carlo evidence on the quasi-structural parameters,  $\lambda$ ,  $\gamma_f$ , and  $\gamma_b$ . According to the theoretical model, these parameters satisfy the restrictions  $\gamma_f, \gamma_b \geq 0$ ,  $\gamma_f + \gamma_b < 1$  and  $\lambda \geq 0$  (see Buiters and Jewitt, 1989, and Galí and Gertler, 1999)—the reduced-form is then determinate.<sup>12</sup> The vector of parameters,  $(\gamma_f, \gamma_b, \lambda)$ , is, respectively, given by: (0.650, 0.300, 0.100) and (0.850, 0.100, 0.100). The former is our benchmark.<sup>13</sup> For each parameter combination, the autoregressive parameters are set to  $\rho_1 = .9(1 - \rho_2)$  and  $\rho_2 = -.65$ . The error terms  $\epsilon_t$  and  $v_t$  are drawn from a bivariate normal distribution with standard deviations  $\sigma_\epsilon = .05$  and  $\sigma_v = .4$ . The correlation coefficient,  $\rho$ , takes, respectively, the values -0.5, 0, and 0.5.

To investigate how the number of instruments affects the performance of the estimators, we consider different numbers of instruments,  $q$ . Each instrument set includes  $q/2$  lags of  $y_t$  and  $x_t$  where  $q$  equals, respectively, 4, 8, 12 and 16. Regarding the sample size, we consider either a small sample with 160 observations—a sample size often encountered in applied macro works (e.g., Kurmann, 2005;

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<sup>11</sup>The derivation of both the concentration parameter and the reduced-form model is well known in the case of our DGP. Both are stated in Appendix 3 for completeness.

<sup>12</sup>We also consider the case in which  $\gamma_f + \gamma_b = 1$ . Following Blanchard and Kahn (1980), two situations can be encountered. When  $\gamma_f \leq 0.5$ , the solution of the characteristic polynomial is unique, but  $y_t$  is a non-stationary process regardless the dynamics of  $x_t$ . When  $\gamma_f > 0.5$  and second-order stationary conditions on the forcing variable hold true, the existence of a stationary solution is guaranteed, but there are in fact infinitely many solutions characterized by sunspot shocks. Results are not reported here but are available upon request.

<sup>13</sup>As a robustness check, the last table (Table 7) also provides evidence in two other cases: (0.550, 0.300, 0.100) and (0.750, 0.100, 0.100).

Nason and Smith, 2008; Rudd and Whelan, 2005 and 2006)—or a large sample with 500 observations. All results reported below are based on 5,000 simulations. For each repetition, we determine the CUE, the 2S-GMM, the smoothed 3S-EEL, the smoothed 3SW-EEL estimators of the quasi-reduced form parameters  $(\gamma_b, \gamma_f, \lambda)$ .<sup>14</sup> We then calculate the median bias and the root mean squared error (RMSE) of the estimators over the 5,000 samples.<sup>15</sup>

Table 1 reports the small and large sample simulation results (T=160 and 500) for both the benchmark parameter vector (0.650, 0.300, 0.100) and the combination (0.850, 0.100, 0.100) in the absence of correlation between the shocks ( $\rho = 0$ ). In both cases the concentration parameter provides evidence that the parameters are well-identified irrespective of the sample size in our benchmark case.

[Insert Table 1 around here]

With regard to each estimator, our results lead to the following interpretation. Firstly, the 2S-GMM estimator is clearly dominated by other estimators in finite and large samples. More specifically, the median bias of the 2S-GMM estimator is lower than those of the proposed smoothed three-step estimators and the CUE in 4 out of the 48 cases in Table 1. It turns out that the median bias differences between the 2S-GMM estimator and the other estimators are significant—the main exception being the case of a small number of instruments. Indeed, these differences are captured by the value of both the intrinsic persistence inherited from the reduced-form coefficient of the lagged endogenous variable  $y_{t-1}$  and the value of the extrinsic persistence measured by the reduced-form coefficient of the forcing variable. As reported in Appendix 3, both coefficients depend critically on the estimates of  $\gamma_f$ ,  $\gamma_b$ , and  $\lambda$ . Finally, in the case of the RMSE, the proposed estimators (respectively, the CUE) outperform the 2S-GMM estimators in 41 (respectively, 31) of the 48 cases.

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<sup>14</sup> From a computational view, we use the numerical optimization routine *fminsearch.m*, which is a part of the "Optimization toolbox" in Matlab. We discard cases where the routine failed to converge. For consistency, when the routine failed to converge for one set of instruments (for a given parameter vector), more samples were generated to compensate for those convergence failures. At the end, all results are based on the same number of repetitions and are comparable across the instrument sets (for a given parameter vector). Initial values were set to the true ones. While the smoothed 3S-EEL and the 2S-GMM estimators were immune to such an initialization, the CUE often fails to converge in 2% to 6% of the cases considered in our simulation experiments or yields large implausible values of the parameters. This numerical instability of GEL-based estimators is well-known in the literature and has been documented among others by Guggenberger and Hahn (2005), and Anderson and Kunitomo (2005). Consequently, the CUE may display higher (mean) median bias and RMSE, especially in finite samples.

<sup>15</sup>The median absolute deviation was also calculated. Results are not reported here but are available upon request.

Secondly, the proposed smoothed three-step estimators compete favorably with the CUE in both finite and large samples. In particular, we observe that the CUE has a larger RMSE than the proposed smoothed three-step estimators in finite samples. This pattern is even more pronounced for the forcing variable coefficient. In contrast, the proposed estimators are dominated by the CUE in terms of median bias, especially as the number of instruments is greater than or equal to 12 (see further). Finally, as the sample size increases, the CUE slightly outperforms the proposed estimators in terms of RMSE but only for the benchmark case.

Thirdly, among the proposed smoothed three-step estimators, the smoothed 3SW-EEL generally outperforms the time-series extension of the 3S-EEL in our benchmark. The same conclusion holds for the large sample. In contrast, results are less clear cut when the DGP is mostly forward-looking (see further). The smoothed 3SW-EEL estimator yields a lower median bias in finite samples than the smoothed 3S-EEL estimator. It is however at the cost of a larger RMSE. As the sample size increases, it depends on the parameter of interest. Overall, as the number of instruments increases, the median bias of the smoothed 3S-EEL estimator tends to grow faster than the one of the 3SW-EEL estimator.

Fourthly, regarding the computational burdensome, the smoothed 3S-EEL estimator is less demanding than the other estimators as to be expected. Notably the better finite and large sample median bias properties of the smoothed 3SW-EEL are obtained at the expense of a slightly higher but tractable computation time. Finally, the role of the shrinkage is an important issue to assess the finite sample performances of the two proposed estimators. Regarding Table 1, both estimators rely more on the shrinkage procedure when (i) the number of instruments is large, and (ii) the dynamics is more forward-looking—this is also the case when the concentration parameter suggests a weak identification problem (see further). Notably, in the case of the smoothed 3S-EEL estimator, the percentage use of the shrinkage procedure in the benchmark case is respectively 12.4% (4 instruments), 65.4% (8 instruments), 91.9% (12 instruments) and 97.7% (16 instruments). On the other hand, when the dynamics is more forward-looking, these percentages are respectively 19.4% (4 instruments), 71.8% (8 instruments), 93.5% (12 instruments) and 97.9% (16 instruments). Unsurprisingly, the smoothed

3SW-EEL estimator rests even more on the shrinkage procedure in both cases, especially for a small set of instruments—the percentages being of the same magnitude when the dimension of the instruments vector increases—since the smoothed implied probabilities that intervene in the reweighted Jacobian estimate are reevaluated during the third step (Definition 2). All in all, the observed relation between the use of the shrinkage procedure and the number of instruments might explained why the median bias performances of the two proposed estimators get worse relative to the CUE as  $q$  equals 12 and 16.

To gain further intuition about the behavior of the proposed estimators, Table 2 summarizes results on the 5%, 25%, 50%, 75% and 95% quantiles for the two parameter combinations in Table 1. To save space, we only report the quantiles of  $\gamma_f$  and  $\lambda$ . While the quantiles of the 2S-GMM estimator and CUE tend to be very different, Monte Carlo results show that those of the proposed smoothed three-step estimators and the CUE are much closer in both finite and large samples. In particular, higher-order theory is reflected in large samples.

[Insert Table 2 around here]

We now assess the robustness of our simulation results with respect to the correlation parameter. Table 3 reports the Monte Carlo results when the two error terms are positively correlated ( $\rho = 0.5$ ). Three points are worth commenting. First, the estimators are more median-biased than in Table 1, especially as the number of instruments increases. In particular, all estimators display a medium to large bias on  $\gamma_b$  and  $\lambda$  when the model is estimated with 12 or more instruments. In that respect, the ranking of the estimators does not convey so much information for empirical applications with a large set of instruments, as long as the sample size is too small. It suggests, if anything, that the model might be more consistently estimated with a small number of instruments when the two estimated shocks are positively correlated.

[Insert Table 3 around here]

Second, the 2S-GMM estimator is still dominated by the proposed three-step estimators in terms of both bias and RMSE, irrespective of the sample size and the parameter combination. As shown

in Table 4, the study of the quantiles leads to the same conclusion. Third, the smoothed 3SW-EEL estimator ought to be preferred to the CUE in finite and large samples when the number of instruments is less than or equal to 12. In the case of 16 instruments, the CUE is less median-biased at the expense of a larger RMSE relative to the smoothed three-step estimators. On the other hand, as in Table 1, the CUE outperforms other estimators in large samples with many moment conditions ( $q \geq 16$ ). In support of this claim, unreported results with 24 instruments strongly favor this interpretation. Unsurprisingly, it is consistent with the higher-order asymptotic properties derived by Newey and Smith (2004), and Anatolyev (2005).<sup>16</sup>

[Insert Table 4 around here]

When the error terms are negatively correlated (Table 5), the size and the sign of the (median) bias of each parameter are close to the ones reported in Table 1. Accordingly, both the RMSE and the bias results of the 2S-GMM estimator are generally higher than those of other estimators in our benchmark, with the exception of a few cases. Regarding the proposed estimators, they perform well relative to the CUE, especially when the number of instruments is small ( $q = 4$  and  $8$ ). For  $q \geq 12$ , the three-step based estimators are between the 2S-GMM estimator and the CUE, which may reduce to some extent their attractiveness. However, the superior finite sample median bias properties of the CUE are again obtained at the cost of both a higher RMSE for  $\gamma_f$  and  $\gamma_b$ , and a larger upper tail for the forcing variable coefficient (see Table 6). Interestingly, the smoothed 3SW-EEL also performs well relative to the CUE in large samples.

[Insert Table 5 around here]

On the other hand, when the DGP is mostly forward-looking, the smoothed three-step estimators generally outperform other estimators in terms of RMSE, irrespective of the sample size. At the same time, the median bias comparison between the 3S-EEL and the 3SW-EEL estimators is less clear and depends, as in Table 1, on the coefficient of interest, the number of instruments, and the sample size. Finally, as the number of instruments increases, the CUE has generally better finite sample median bias properties for all coefficients of interest at the cost of a higher RMSE, especially for the forcing

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<sup>16</sup>Note however that the bias result of Newey and Smith (2004) and Anatolyev (2005) is about the higher-order mean bias, whereas our findings is about the median bias. Unreported mean bias results still confirm our claim.

variable coefficient. Moreover, the CUE does not always prevail asymptotically over the proposed estimators: it depends on the parameter of interest and the number of instruments.

[Insert Table 6 around here]

To assess the robustness of our results, we conduct an extensive study of both the bias and RMSE performances of the previous estimators. In doing so, we reevaluate the median (mean) bias and the RMSE of all estimators using all values of  $\gamma_f$  between 0 and 0.95 with an increment of 0.0125. For each DGP, the value of  $\gamma_b$  (respectively,  $\lambda$ ) is defined to be  $1 - \gamma_f$  (respectively, 0.1). To save space, we report the finite sample results ( $T = 160$ ) in the absence of correlation and for  $q = 8, 12$  and 16. The conclusions remain roughly the same in other cases which are available upon request. Results are reported in Figures 1-3.

Several interesting results are worth discussing. First, regarding the sign of each bias, all estimators generally underestimate the forward-looking contribution, with the exception of predominantly backward-looking DGPs.<sup>17</sup> In contrast, the bias sign is less clear for the backward-looking coefficient. More specifically, as the number of instruments increases and the DGP is mostly driven by the forward-looking component, the backward-looking parameter is overestimated. The converse is true when the DGP is mostly backward-looking. Finally, the contribution of the forcing variable is generally overestimated when the dynamics of  $y_t$  is not too forward-looking. For large values of  $\gamma_f$ , all estimators underestimate  $\lambda$ . As a result, when the true DGP is mostly forward-looking, all estimators favor in finite samples, if anything, an hybrid representation in which the inertia of the dependent variable ( $\gamma_b$ ) is spurious and the relevance of the forcing variable is downsized. These results are robust irrespective of the correlation coefficient  $\rho$ . Second, the size of the median (mean) bias is far from being negligible when the DGP is either mostly backward-looking (respectively, forward-looking)—with the exception of the forcing variable coefficient—or the true parameter value of  $\gamma_f$  is approximatively in the interval (0.45, 0.60). Unsurprisingly, the same patterns are observed for the RMSE. The (inverted) bell-shaped of the estimates around  $\gamma_f = 0.5$  can be explained by the value of the concentration parameter (Figure

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<sup>17</sup>In finite samples, this fact might be explained by the information content (relevance) of the set of instruments. Indeed as  $\gamma_f$  increases toward one, the relevance of the past values of  $y_t$  is weaker and weaker, and the dynamics depends more and more on future values of the forcing variable. The picture is different when the dynamics is mostly backward-looking—the key issue might be the redundancy of the instruments.

4), which suggests a weak identification problem, especially in the case of finite samples.<sup>18</sup> Interestingly, it depends on the estimator and the number of moment conditions. All in all, this result is consistent with the evidence reported in Dufour et al. (2006) and Mavroeidis (2005).

[Insert Figures 1-4 around here]

Regarding the relative performance of each estimator, we first note that the 2S-GMM is generally outperformed by other estimators in terms of median (mean) bias, with the exception of a small number of instruments ( $q = 4$ ). On the other hand, the RMSE of the 2S-GMM estimator compares very favorably with respect to the CUE in finite samples irrespective of the number of moment conditions. Second, the median (mean) bias properties of the CUE are better than those of other estimators for all coefficients, especially when the number of instruments increases. Two points might explain (at least) this behavior. On the one hand, the two proposed estimators rely more on the shrinkage procedure in finite samples as the dimension of the instruments set increases, which might cause worse (median) bias performances. On the other hand, the CUE is less sensitive to the weak identification problem.<sup>19</sup> At the same time, these better (median) bias properties of the CUE are generally obtained at the expense of both more extreme parameter estimates and thus a larger RMSE of  $\gamma_f$  and  $\gamma_b$  as the number of moment conditions grows, and a very imprecise variance estimate for the forcing variable coefficient. Third, the proposed smoothed three-step estimators perform very well with respect to the 2S-GMM estimator, except to some extent for a nearly just-identified model ( $q = 4$ ). As the number of moments conditions increases ( $q \geq 16$ ), the finite sample median (mean) bias performances of these estimators are reduced relative to the CUE—they are close to those of the 2S-GMM estima-

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<sup>18</sup>To investigate further the weak identification problem, we also conduct simulations in which the autoregressive parameters of the forcing variable,  $\rho_1$  and  $\rho_2$ , are such that the DGP is weakly identified irrespective of the sample size and the parameter combination. In that respect, the corresponding autoregressive parameters are now given by  $\rho_1 = 0.9(1 - \rho_2)$  and  $\rho_2 = -0.65/\sqrt{T}$ . Weak identification arises here from the forcing variable DGP, i.e. past values of the forcing variable are weak instruments. Consequently, all estimators are significantly biased irrespective of the correlation parameter and the parameter combination. Second, the RMSE also significantly increases for all estimators relative to the corresponding well-identified case. Moreover, the bias and the RMSE increase with the number of instruments since instruments are weak and thus do not convey reliable information. Finally, as to be expected from theory, the RMSE and the median bias do not significantly fall with the sample size, i.e. the estimators do not converge to their true values. These results are in line with those of Stock and Wright (2000), Kleibergen (2002) and Mavroeidis (2004).

<sup>19</sup>From a numerical point of view, two points are also worth noting: (i) We exclude without any penalization the cases in which the numerical procedure of the CUE fails to converge (see footnote 14) and (ii) The computational burden of the CUE might be reduced (even if not negligible in the reported Monte Carlo simulations) because only a univariate DGP is considered.

tor.<sup>20</sup> However their finite sample RMSE performances are generally preferable to those of the CUE and the 2S-GMM estimator. All in all, the smoothed 3SW-EEL estimator generally dominates the smoothed 3S-EEL estimator in terms of median bias. The interpretation is less clear for the RMSE. The smoothed 3S-EEL estimator displays better finite sample RMSE performances than the smoothed 3SW-EEL estimator when the number of instruments is small ( $q \leq 8$ ). The converse is true when the number of moment conditions is large. On the other hand, the smoothed 3SW-EEL estimator behaves asymptotically more likely as the CUE for both criteria, i.e. the smoothed 3SW-EEL is consistent with higher-order asymptotics theory. These interpretations remain valid irrespective of the correlation coefficient.

To summarize, our Monte Carlo simulations provide evidence that the proposed estimators, the smoothed 3S-EEL and the smoothed 3SW-EEL, compare extremely favorably with respect to the 2S-GMM estimator in terms of both (mean) median bias and RMSE. Second, both smoothed estimators perform well with respect to the CUE in finite and large samples, especially as there are not too many moment restrictions ( $q \leq 12$ ). For medium to large instrument sets, the CUE is less median-biased in finite samples but at the expense of both a higher RMSE and heavy tails. Inconsistent with the higher-order theory, the (median) bias of the CUE can be larger than those of the proposed estimators in large samples. This may be a consequence of the computational burden resulting from both the saddle point characterization of the CUE and its numerical instability. Third, among the proposed smoothed 3S-EEL estimators, the smoothed 3SW-EEL estimator has generally better finite and large sample median (mean) bias properties than the time-series extension of the 3S-EEL estimator when the forward-looking component does not overrule the backward-looking one. When the DGP is mostly forward-looking, results are more mixed and depend on the parameter of interest, the correlation parameter and the number of instruments. On the other hand, the smoothed 3S-EEL estimator performs very well in terms of RMSE, especially as the number of instruments is small. Fourth, the finite sample bias encountered in univariate rational expectations models might be substantial even if the DGP is well-identified. For example, when the true DGP is nearly a purely forward-looking process (i.e.,  $\gamma_f$

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<sup>20</sup>As the sample size increases ( $T = 500$ ), the differences with the 2S-GMM estimators remain significant and there is a gain to use the three-step estimators.

is near one), all estimators favor an hybrid specification in which the backward-looking component is spurious and the contribution of the forcing variable is underestimated. All in all, this may significantly distort estimates and thus the corresponding interpretation in terms of structural parameters. Fifth, as to be expected, the smoothed 3S-EEL estimator is less computationally demanding than the other estimators (irrespective of the use of the shrinkage procedure), especially with respect to the CUE. This remains true even when using the shrinkage procedure. The smoothed 3SW-EEL remains computationally convenient at the expense of a higher but tractable computation time. Sixth, the use of the shrinkage procedure is a key issue for the finite sample performances of the two proposed estimators. Notably, both estimators rely more on the shrinkage procedure in finite samples when (i) the number of instruments is large, (ii) the dynamics is more forward-looking and (iii) the concentration parameter suggests a weak identification problem

## 5 Conclusion

Using Bonnal and Renault (2001) and Antoine, Bonnal and Renault (2007), we study two smoothed three-step EEL-based estimators for weakly dependent data. Both estimators achieve a higher-order equivalence to the SEL (up to an order  $\mathcal{O}_p((2K_T + 1)/T^{3/2})$ ). In addition, these estimators are more computationally convenient than the ones of the class of (smoothed) GEL estimators.

A Monte Carlo study reveals that the finite sample properties of the proposed estimators are competitive with respect to the 2S-GMM estimator and the CUE. Moreover, among the proposed smoothed three-step estimators, the smoothed 3SW-EEL estimator generally has better finite and large sample median (mean) bias properties than the time-series extension of the 3S-EEL estimator at the expense of a slightly higher computational cost. On the other hand, the smoothed 3S-EEL estimator performs very well in terms of RMSE, especially as the number of instruments is small.

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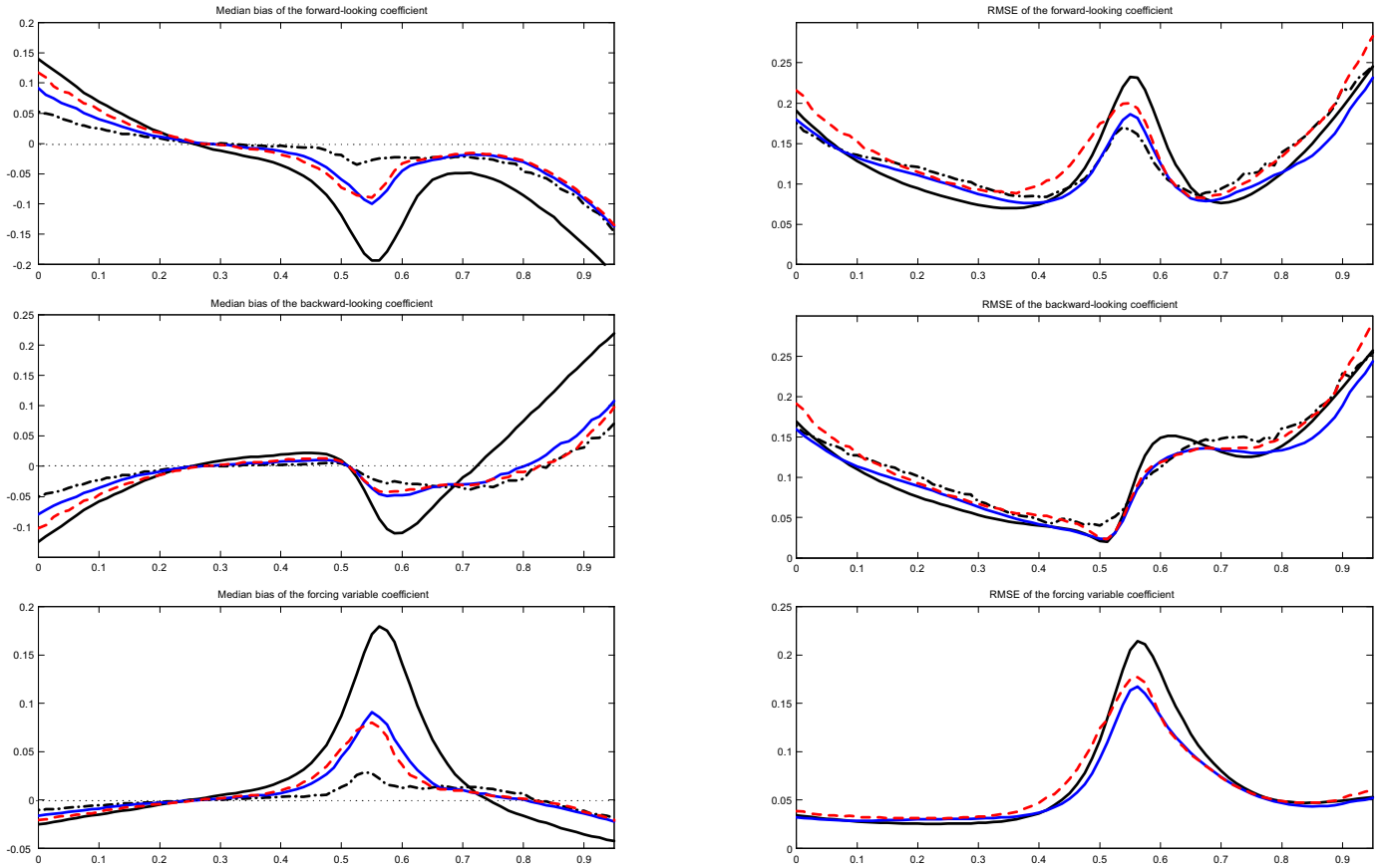
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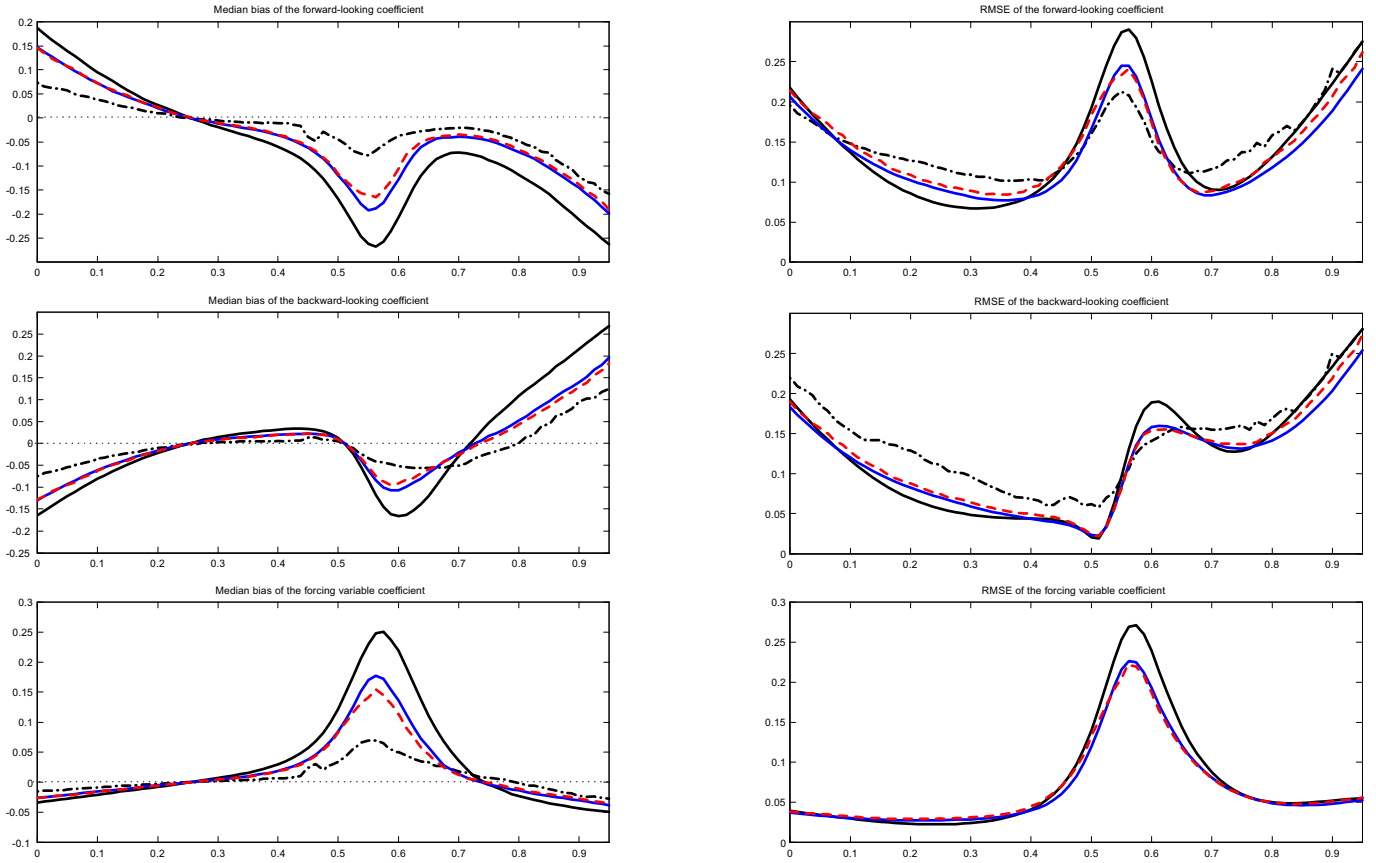
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**Figure 1:** Monte Carlo simulations with 8 instruments



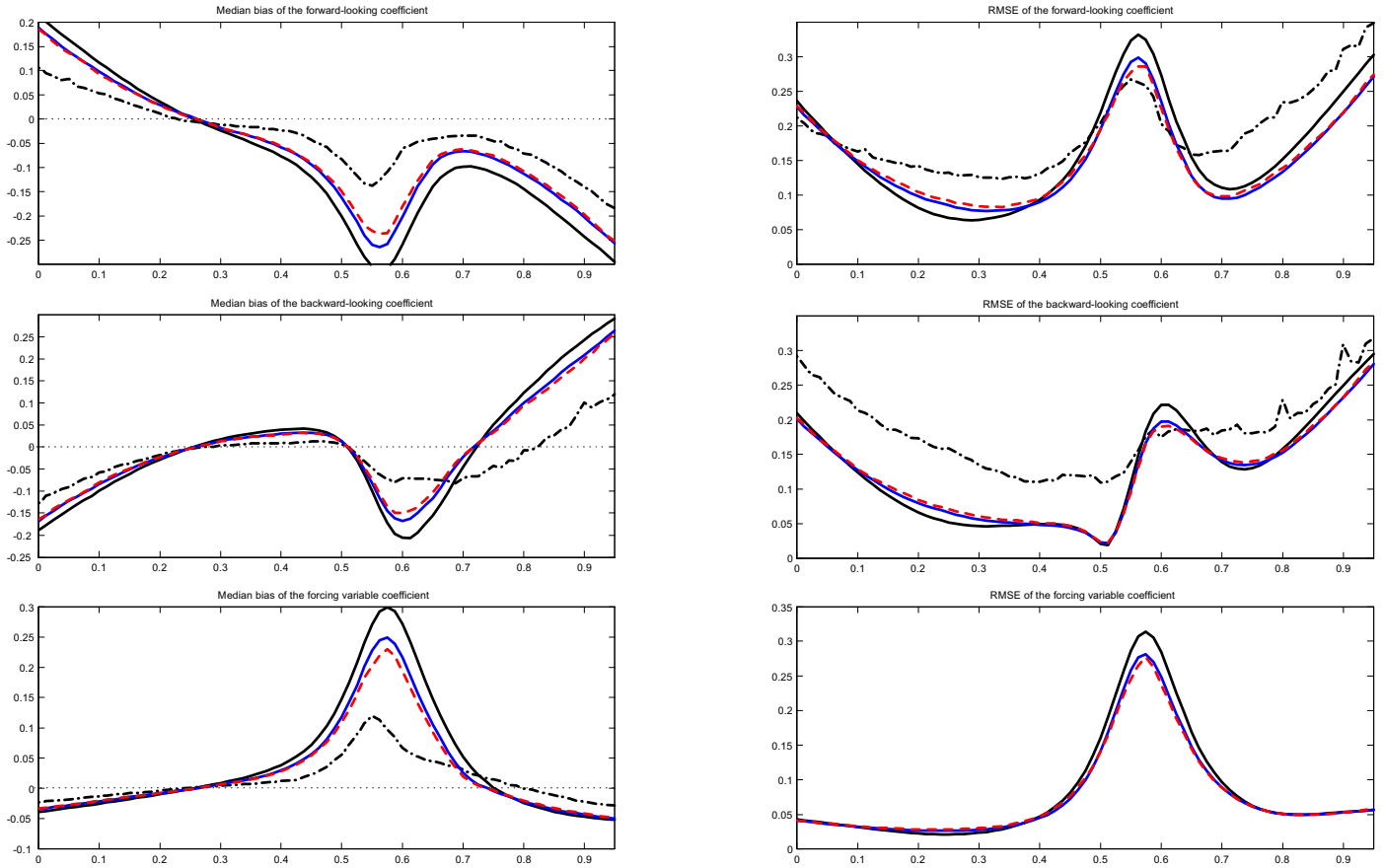
The solid black, dotted black, dotted red, and blue lines represent, respectively, the median bias or RMSE of the 2S-GMM estimator, the CUE, the smoothed 3SW-EEL estimator, and the smoothed 3S-EEL estimator. The x-axis is the value of the forward-looking coefficient  $\gamma_f$ . The left and right panels are respectively the median bias and the RMSE of the forward-looking, backward-looking, and forcing variable coefficients. For sake of presentation, the RMSE of the CUE is not reported when it significantly exceeds those of other estimators. The sample size is 160 and the correlation coefficient is zero.

**Figure 2:** Monte Carlo simulations with 12 instruments



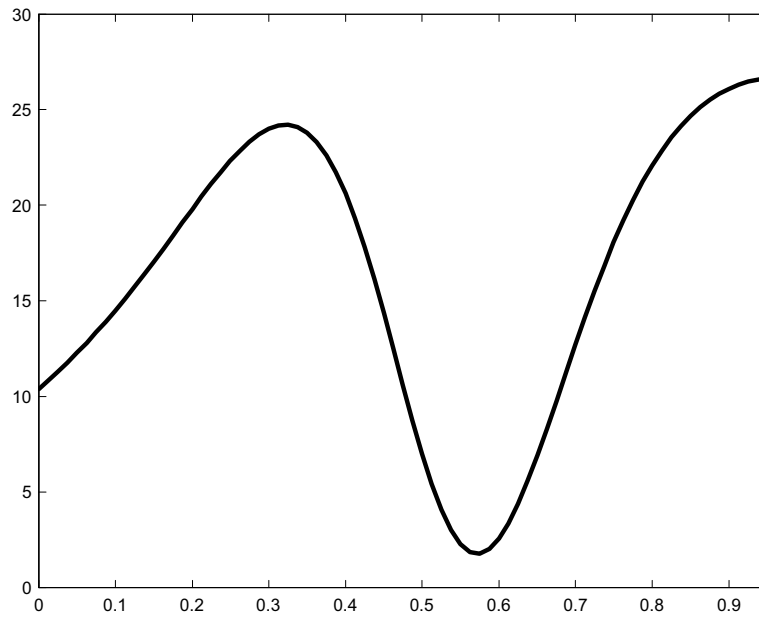
The solid black, dotted black, dotted red, and blue lines represent, respectively, the median bias or RMSE of the 2S-GMM estimator, the CUE, the smoothed 3SW-EEL estimator, and the smoothed 3S-EEL estimator. The x-axis is the value of the forward-looking coefficient  $\gamma_f$ . The left and right panels are respectively the median bias and the RMSE of the forward-looking, backward-looking, and forcing variable coefficients. For sake of presentation, the RMSE of the CUE is not reported when it significantly exceeds those of other estimators. The sample size is 160 and the correlation coefficient is zero.

**Figure 3:** Monte Carlo simulations with 16 instruments



The solid black, dotted black, dotted red, and blue lines represent, respectively, the median bias or RMSE of the 2S-GMM estimator, the CUE, the smoothed 3SW-EEL estimator, and the smoothed 3S-EEL estimator. The x-axis is the value of the forward-looking coefficient  $\gamma_f$ . The left and right panels are respectively the median bias and the RMSE of the forward-looking, backward-looking, and forcing variable coefficients. For sake of presentation, the RMSE of the CUE is not reported when it significantly exceeds those of other estimators. The sample size is 160 and the correlation coefficient is zero.

**Figure 4:** Concentration parameter



The determination of the concentration parameter is based on the reduced-form and is explained in Appendix 3. Large values (respectively, small values) of the concentration parameter support evidence that the model is well-identified (respectively, weakly-identified). The x-axis is the value of the forward-looking coefficient  $\gamma_f$ .

**Table 1:** Monte Carlo simulations

$T = 160$		$\gamma_f = 0.650$		$\gamma_b = 0.300$		$\lambda = 0.100$		$\gamma_f = 0.850$		$\gamma_b = 0.100$		$\lambda = 0.100$	
Estimators	Inst.	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
2S-GMM	4	-0.0271	0.0744	-0.0287	0.1260	0.0175	0.0915	-0.0604	0.1258	0.0226	0.1422	-0.0038	0.0420
3S-EEL	4	-0.0159	0.0747	-0.0251	0.1233	0.0099	0.0862	-0.0186	0.1416	-0.0402	0.1498	0.0060	0.0428
3SW-EEL	4	-0.0160	0.0770	-0.0257	0.1234	0.0093	0.0865	-0.0223	0.1813	-0.0348	0.1854	0.0054	0.0489
CUE	4	-0.0186	0.0752	-0.0241	0.1241	0.0097	0.0875	-0.0305	0.1380	-0.0328	0.1494	0.0032	0.0424
2S-GMM	8	-0.0664	0.0981	-0.0703	0.1470	0.0668	0.1195	-0.1257	0.1514	0.1170	0.1691	-0.0243	0.0467
3S-EEL	8	-0.0318	0.0842	-0.0374	0.1349	0.0233	0.0999	-0.0620	0.1394	0.0208	0.1525	-0.0042	0.0440
3SW-EEL	8	-0.0280	0.0861	-0.0351	0.1344	0.0193	0.0994	-0.0560	0.1646	0.0105	0.1743	-0.0021	0.0473
CUE	8	-0.0244	0.0893	-0.0377	0.1402	0.0180	0.1660	-0.0504	0.1646	0.0073	0.1779	-0.0031	> 1
2S-GMM	12	-0.1057	0.1269	-0.1171	0.1693	0.1135	0.1461	-0.1662	0.1772	0.1644	0.1901	-0.0343	0.0496
3S-EEL	12	-0.0581	0.1013	-0.0626	0.1530	0.0573	0.1215	-0.1149	0.1510	0.1057	0.1686	-0.0236	0.0469
3SW-EEL	12	-0.0501	0.1015	-0.0536	0.1508	0.0426	0.1185	-0.1074	0.1636	0.0962	0.1785	-0.0205	0.0485
CUE	12	-0.0302	0.1085	-0.0490	0.1595	0.0296	> 1	-0.0795	0.1934	0.0428	0.2041	-0.0094	> 1
2S-GMM	16	-0.1434	0.1563	-0.1557	0.1892	0.1480	0.1709	-0.1916	0.1986	0.1853	0.2013	-0.0360	0.0502
3S-EEL	16	-0.0952	0.1296	-0.1065	0.1759	0.1003	0.1493	-0.1569	0.1741	0.1624	0.1898	-0.035	0.0500
3SW-EEL	16	-0.0850	0.1266	-0.0969	0.1735	0.0891	0.1454	-0.1527	0.1791	0.1562	0.1937	-0.0334	0.0506
CUE	16	-0.0417	0.1480	-0.0902	0.1841	0.0562	> 1	-0.1112	0.2437	0.0855	0.2442	-0.0180	> 1
$T = 500$													
2S-GMM	4	-0.0090	0.0418	-0.0111	0.0724	0.0042	0.0529	-0.0183	0.0749	0.0057	0.0952	-0.0020	0.0284
3S-EEL	4	-0.0046	0.0419	-0.0087	0.0705	-0.0000	0.0510	0.0042	0.0789	-0.0251	0.0877	0.0053	0.0258
3SW-EEL	4	-0.0045	0.0418	-0.0084	0.0703	-0.0000	0.0509	0.0051	0.0817	-0.0293	0.0895	0.0063	0.0262
CUE	4	-0.0060	0.0413	-0.0082	0.0710	0.0008	0.0513	-0.0070	0.0772	-0.0109	0.0943	0.0017	0.0277
2S-GMM	8	-0.0210	0.0479	-0.0238	0.0809	0.0189	0.0615	-0.0505	0.0831	0.0554	0.1071	-0.0137	0.0309
3S-EEL	8	-0.0067	0.0432	-0.0093	0.0728	-0.0003	0.0528	-0.0029	0.0813	-0.0209	0.0917	0.0035	0.0269
3SW-EEL	8	-0.0058	0.0431	-0.0088	0.0726	-0.0009	0.0525	0.0006	0.0831	-0.0295	0.0920	0.0052	0.0268
CUE	8	-0.0077	0.0426	-0.0092	0.0743	0.0014	0.0535	-0.0119	0.0802	-0.0073	0.0977	0.0002	0.0285
2S-GMM	12	-0.0347	0.0564	-0.0365	0.0894	0.0344	0.0709	-0.0786	0.0976	0.0874	0.1228	-0.0214	0.0336
3S-EEL	12	-0.0105	0.0453	-0.0125	0.0763	0.0036	0.0562	-0.0156	0.0843	0.0018	0.0978	-0.0020	0.0285
3SW-EEL	12	-0.0090	0.0449	-0.0111	0.0755	0.0020	0.0553	-0.0085	0.0857	-0.0126	0.0972	0.0016	0.0282
CUE	12	-0.0083	0.0441	-0.0125	0.0772	0.0043	0.0556	-0.0167	0.0833	-0.0020	0.1015	-0.0008	0.0293
2S-GMM	16	-0.0485	0.0659	-0.0523	0.0999	0.0497	0.0815	-0.0986	0.1123	0.1160	0.1383	-0.0282	0.0365
3S-EEL	16	-0.0170	0.0500	-0.0165	0.0845	0.0110	0.0637	-0.0355	0.0888	0.0336	0.1059	-0.0101	0.0301
3SW-EEL	16	-0.0146	0.0493	-0.0143	0.0832	0.0080	0.0623	-0.0259	0.0897	0.0112	0.1046	-0.0050	0.0298
CUE	16	-0.0102	0.0460	-0.0163	0.0822	0.0083	0.0594	-0.0192	0.0883	0.0043	0.1068	-0.0022	0.0302

Note: For each parameter combination, the correlation parameter equals 0. For  $\gamma_f = 0.650$ ,  $\gamma_b = 0.300$  and  $\lambda = 0.100$ , the concentration parameter equals, respectively, 6.89 ( $T = 160$ ) and 21.36 ( $T = 500$ ). For  $\gamma_f = 0.850$ ,  $\gamma_b = 0.100$  and  $\lambda = 0.100$ , the concentration parameter equals, respectively, 24.67 ( $T = 160$ ) and 76.47 ( $T = 500$ ). The variance-covariance matrix of the moment conditions is estimated using the automatic lag selection procedure of Newey and West (1994). The number of simulations is 5,000.

Table 2: Monte Carlo simulations—percentiles

$T = 160$	Estimators	Inst.	$\gamma_f = 0.650$					$\lambda = 0.100$					$\gamma_f = 0.850$					$\lambda = 0.100$							
			5%	25%	50%	75%	95%	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%			
2S-GMM	4	2S-GMM	0.5110	0.5793	0.6229	0.6633	0.7356	0.0544	0.0766	0.1175	0.1998	0.3043	0.6212	0.7247	0.7896	0.8708	1.0000	0.0308	0.0630	0.0962	0.1285	0.1605			
		3S-EEL	0.5226	0.5903	0.6341	0.6782	0.7616	0.0556	0.0774	0.1099	0.1864	0.2946	0.6357	0.7531	0.8314	0.9299	1.0000	0.0349	0.0758	0.1060	0.1300	0.1603			
		3SW-EEL	0.5212	0.5899	0.6340	0.6787	0.7627	0.0561	0.0773	0.1093	0.1859	0.2959	0.5888	0.7489	0.8277	0.9287	1.0000	0.0227	0.0747	0.1054	0.1292	0.1591			
		CUE	0.5209	0.5881	0.6314	0.6748	0.7555	0.0548	0.0757	0.1097	0.1894	0.2969	0.6325	0.7418	0.8195	0.9177	1.0000	0.0339	0.0690	0.1032	0.1297	0.1602			
2S-GMM	8	2S-GMM	0.4681	0.5327	0.5836	0.6297	0.6993	0.0561	0.0944	0.1668	0.2452	0.3406	0.5957	0.6701	0.7243	0.7905	0.9090	0.0229	0.0500	0.0757	0.1139	0.1599			
		3S-EEL	0.4962	0.5714	0.6182	0.6668	0.7484	0.0534	0.0771	0.1233	0.2080	0.3194	0.6170	0.7134	0.7880	0.8890	1.0000	0.0270	0.0609	0.0958	0.1261	0.1594			
		3SW-EEL	0.4942	0.5738	0.6220	0.6692	0.7549	0.0529	0.0766	0.1193	0.2034	0.3219	0.5889	0.7144	0.7940	0.8969	1.0000	0.0204	0.0627	0.0979	0.1268	0.1573			
		CUE	0.5049	0.5783	0.6256	0.6701	0.7591	0.0529	0.0766	0.1180	0.2051	0.3202	0.5984	0.7147	0.7996	0.9040	1.0000	0.0237	0.0628	0.0969	0.1272	0.1600			
2S-GMM	12	2S-GMM	0.4302	0.4935	0.5443	0.5920	0.6595	0.0595	0.1340	0.2135	0.2844	0.3638	0.5779	0.6373	0.6838	0.7388	0.8357	0.0193	0.0425	0.0657	0.1036	0.1610			
		3S-EEL	0.4538	0.5367	0.5919	0.6412	0.7236	0.0543	0.0855	0.1573	0.2500	0.3487	0.5896	0.6713	0.7351	0.8192	0.9915	0.0224	0.0485	0.0764	0.1163	0.1588			
		3SW-EEL	0.4577	0.5431	0.5999	0.6475	0.7354	0.0531	0.0802	0.1426	0.2433	0.3461	0.5758	0.6699	0.7426	0.8332	1.0000	0.0182	0.0488	0.0795	0.1186	0.1590			
		CUE	0.4830	0.5699	0.6198	0.6707	0.7822	0.0504	0.0776	0.1296	0.2276	0.3354	0.5570	0.6780	0.7705	0.8937	1.0000	0.0132	0.0515	0.0906	0.1269	0.1644			
2S-GMM	16	2S-GMM	0.4004	0.4621	0.5066	0.5583	0.6221	0.0728	0.1774	0.2480	0.3195	0.3808	0.5640	0.6193	0.6584	0.7024	0.7809	0.0203	0.0405	0.0640	0.1029	0.1592			
		3S-EEL	0.4184	0.4927	0.5548	0.6093	0.6901	0.0559	0.1095	0.2003	0.2982	0.3727	0.5720	0.6407	0.6931	0.7601	0.9073	0.0199	0.0427	0.0665	0.1052	0.1582			
		3SW-EEL	0.4174	0.4986	0.5650	0.6177	0.7024	0.0538	0.0998	0.1891	0.2913	0.3734	0.5571	0.6397	0.6973	0.7743	0.9429	0.0180	0.0427	0.0666	0.1078	0.1589			
		CUE	0.4117	0.5467	0.6083	0.6665	0.8102	0.0478	0.0831	0.1562	0.2634	0.3824	0.2970	0.6460	0.7388	0.8746	1.0000	-	0.0210	0.0431	0.082	0.1247	0.1734		
$T = 500$																									
2S-GMM	4	2S-GMM	0.5701	0.6136	0.6410	0.6638	0.7043	0.0637	0.0768	0.1042	0.1485	0.2153	0.7233	0.7816	0.8317	0.8854	0.9681	0.0558	0.0761	0.0980	0.1221	0.1435			
		3S-EEL	0.5737	0.6180	0.6454	0.6702	0.7105	0.0646	0.0772	0.1000	0.1448	0.2123	0.7378	0.8021	0.8542	0.9142	1.0000	0.0626	0.0838	0.1053	0.1239	0.1436			
		3SW-EEL	0.5737	0.6182	0.6455	0.6703	0.7106	0.0647	0.0772	0.1000	0.1448	0.2122	0.7379	0.8026	0.8551	0.9153	1.0000	0.0627	0.0847	0.1063	0.1241	0.1435			
		CUE	0.5734	0.6165	0.6440	0.6680	0.7088	0.0640	0.0766	0.1008	0.1460	0.2126	0.7289	0.7899	0.8430	0.8981	0.9887	0.0575	0.0783	0.1017	0.1232	0.1431			
2S-GMM	8	2S-GMM	0.5559	0.5981	0.6290	0.6553	0.6951	0.0642	0.0806	0.1189	0.1643	0.2320	0.7010	0.7532	0.7995	0.8500	0.9296	0.0492	0.0676	0.0863	0.1140	0.1418			
		3S-EEL	0.5698	0.6143	0.6433	0.6691	0.7114	0.0636	0.0772	0.0997	0.1463	0.2164	0.7280	0.7939	0.8471	0.9080	1.0000	0.0593	0.0803	0.1035	0.1229	0.1437			
		3SW-EEL	0.5701	0.6152	0.6442	0.6695	0.7122	0.0636	0.0769	0.0991	0.1454	0.2161	0.7294	0.7974	0.8506	0.9150	1.0000	0.0608	0.0821	0.1052	0.1241	0.1438			
		CUE	0.5691	0.6148	0.6423	0.6678	0.7091	0.0633	0.0765	0.1014	0.1470	0.2166	0.7181	0.7824	0.8381	0.8980	0.9901	0.0547	0.0763	0.1002	0.1234	0.1434			
2S-GMM	12	2S-GMM	0.5434	0.5840	0.6153	0.6438	0.6852	0.0641	0.0880	0.1344	0.1810	0.2472	0.6823	0.7304	0.7714	0.8215	0.8939	0.0438	0.0616	0.0786	0.1040	0.1392			
		3S-EEL	0.5644	0.6089	0.6395	0.6666	0.7109	0.0632	0.0780	0.1036	0.1532	0.2226	0.7103	0.7774	0.8344	0.8962	1.0000	0.0525	0.0755	0.0980	0.1208	0.1434			
		3SW-EEL	0.5663	0.6113	0.6410	0.6690	0.7114	0.0632	0.0772	0.1020	0.1504	0.2213	0.7114	0.7841	0.8415	0.9056	1.0000	0.0537	0.0777	0.1016	0.1224	0.1437			
		CUE	0.5676	0.6130	0.6417	0.6667	0.7107	0.0629	0.0769	0.1043	0.1505	0.2225	0.7100	0.7770	0.8333	0.8953	0.9928	0.0523	0.0741	0.0992	0.1227	0.1431			
2S-GMM	16	2S-GMM	0.5276	0.5705	0.6015	0.6334	0.6746	0.0661	0.1002	0.1497	0.1983	0.2607	0.6647	0.7110	0.7514	0.7952	0.8679	0.0406	0.0564	0.0718	0.0962	0.1379			
		3S-EEL	0.5554	0.6000	0.6330	0.6616	0.7072	0.0636	0.0789	0.1110	0.1670	0.2384	0.6956	0.7570	0.8145	0.8787	0.9944	0.0487	0.0693	0.0899	0.1157	0.1428			
		3SW-EEL	0.5567	0.6021	0.6354	0.6640	0.7092	0.0633	0.0783	0.1080	0.1639	0.2379	0.6987	0.7649	0.8241	0.8921	1.0000	0.0502	0.0716	0.0950	0.1192	0.1432			
		CUE	0.5645	0.6098	0.6398	0.6671	0.7102	0.0634	0.0775	0.1083	0.1580	0.2290	0.6989	0.7695	0.8308	0.8963	0.9961	0.0488	0.0729	0.0978	0.1204	0.1434			

**Table 3:** Monte Carlo simulations

$T = 160$		$\gamma_f = 0.650$		$\gamma_b = 0.300$		$\lambda = 0.100$		$\gamma_f = 0.850$		$\gamma_b = 0.100$		$\lambda = 0.100$	
Estimators	Inst.	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
2S-GMM	4	-0.0292	0.0794	-0.0382	0.1516	0.0227	0.1115	-0.0782	0.1715	0.0447	0.1894	-0.0102	0.0534
3S-EEL	4	-0.0176	0.0808	-0.0323	0.1476	0.0111	0.1029	-0.032	0.1915	-0.0142	0.2025	0.0039	0.0554
3SW-EEL	4	-0.0183	0.0869	-0.0310	0.1472	0.0102	0.1033	-0.0341	0.2174	-0.0147	0.2252	0.0044	0.0599
CUE	4	-0.0207	0.0815	-0.0303	0.1483	0.0100	0.1038	-0.0458	0.2056	-0.0209	0.2198	0.0001	0.0599
2S-GMM	8	-0.0744	0.1044	-0.1159	0.1799	0.1009	0.1465	-0.1526	0.1834	0.1657	0.2035	-0.0390	0.0552
3S-EEL	8	-0.0345	0.0884	-0.0563	0.1602	0.0327	0.1180	-0.0871	0.1826	0.0713	0.1991	-0.0162	0.0552
3SW-EEL	8	-0.0313	0.0903	-0.0462	0.1580	0.0197	0.1151	-0.0798	0.2087	0.0463	0.2222	-0.0105	0.0595
CUE	8	-0.0286	0.0956	-0.0420	0.1626	0.0231	0.1210	-0.0838	0.2229	0.0648	0.2401	-0.0142	0.0812
2S-GMM	12	-0.1219	0.1366	-0.1861	0.2076	0.1631	0.1793	-0.1954	0.2081	0.2013	0.2212	-0.0468	0.0573
3S-EEL	12	-0.0658	0.1061	-0.1024	0.1808	0.0837	0.1440	-0.1432	0.1836	0.1550	0.2036	-0.0368	0.0554
3SW-EEL	12	-0.0569	0.1053	-0.0715	0.1749	0.0574	0.1372	-0.1321	0.1926	0.1377	0.2098	-0.0321	0.0565
CUE	12	-0.0346	0.1198	-0.0731	0.1801	0.0424	> 1	-0.1258	0.2527	0.1283	0.2695	-0.0303	> 1
2S-GMM	16	-0.1629	0.1677	-0.2475	0.2319	0.2175	0.2081	-0.2162	0.2231	0.2081	0.2206	-0.0435	0.0545
3S-EEL	16	-0.1061	0.1339	-0.1719	0.2049	0.1483	0.1734	-0.1839	0.1997	0.1961	0.2148	-0.0447	0.0557
3SW-EEL	16	-0.0949	0.1332	-0.1537	0.2000	0.1312	0.1672	-0.1786	0.2039	0.1923	0.2185	-0.0430	0.0565
CUE	16	-0.0445	0.1637	-0.1296	0.2053	0.0710	> 1	-0.1554	0.2847	0.1612	0.2907	-0.0387	> 1
$T = 500$													
2S-GMM	4	-0.0112	0.0433	-0.0158	0.0918	0.0053	0.0666	-0.0220	0.0870	0.0104	0.1087	-0.0025	0.0323
3S-EEL	4	-0.0056	0.0433	-0.0087	0.0883	-0.0004	0.0633	-0.0002	0.1003	-0.0131	0.1081	0.0047	0.0304
3SW-EEL	4	-0.0057	0.0433	-0.0096	0.0883	-0.0010	0.0633	0.0011	0.1099	-0.0150	0.1164	0.0056	0.0323
CUE	4	-0.0079	0.0426	-0.0101	0.0892	0.0002	0.0639	-0.0056	0.0906	-0.0173	0.1086	0.0027	0.0317
2S-GMM	8	-0.0241	0.0506	-0.0384	0.1078	0.0300	0.0813	-0.0684	0.1020	0.0849	0.1300	-0.0221	0.0369
3S-EEL	8	-0.0067	0.0448	-0.0106	0.0935	-0.0002	0.0670	-0.0073	0.0995	-0.0072	0.1114	0.0014	0.0316
3SW-EEL	8	-0.0060	0.0449	-0.0113	0.0928	-0.0009	0.0664	-0.0035	0.1097	-0.0113	0.1183	0.0038	0.0327
CUE	8	-0.0087	0.0440	-0.0104	0.0945	0.0011	0.0673	-0.0161	0.0985	0.0030	0.1174	-0.0014	0.0333
2S-GMM	12	-0.0392	0.0608	-0.0649	0.1230	0.0560	0.0959	-0.1079	0.1227	0.1342	0.1538	-0.0343	0.0420
3S-EEL	12	-0.0108	0.0477	-0.0152	0.1010	0.0067	0.0735	-0.0318	0.0997	0.0233	0.1168	-0.0098	0.0334
3SW-EEL	12	-0.0094	0.0474	-0.0118	0.0994	0.0019	0.0719	-0.0200	0.1093	0.0046	0.1224	-0.0026	0.0341
CUE	12	-0.0094	0.0451	-0.0122	0.0990	0.0032	0.0704	-0.0287	0.1069	0.0210	0.1272	-0.0058	0.0357
2S-GMM	16	-0.0546	0.0725	-0.0874	0.1399	0.0741	0.1118	-0.1311	0.1402	0.1608	0.1714	-0.0417	0.0454
3S-EEL	16	-0.0180	0.0525	-0.0244	0.1119	0.0164	0.0834	-0.0599	0.1058	0.0761	0.1290	-0.0216	0.0365
3SW-EEL	16	-0.0153	0.0517	-0.0187	0.1088	0.0110	0.0805	-0.0481	0.1114	0.0514	0.1311	-0.0154	0.0366
CUE	16	-0.0103	0.0472	-0.0165	0.1061	0.0069	0.0755	-0.0387	0.1124	0.0333	0.1340	-0.0095	0.0372

Note: For each parameter combination, the correlation parameter equals 0.5. For  $\gamma_f = 0.650$ ,  $\gamma_b = 0.300$  and  $\lambda = 0.100$ , the concentration parameter equals, respectively, 4.74 ( $T = 160$ ) and 14.67 ( $T = 500$ ). For  $\gamma_f = 0.850$ ,  $\gamma_b = 0.100$  and  $\lambda = 0.100$ , the concentration parameter equals, respectively, 20.86 ( $T = 160$ ) and 64.64 ( $T = 500$ ). The variance-covariance matrix of the moment conditions is estimated using the automatic lag selection procedure of Newey and West (1994). The number of simulations is 5,000.

**Table 4:** Monte Carlo simulations ( $\rho = 0.5$ ) —percentiles

$T = 160$		$\gamma_f = 0.650$										$\gamma_f = 0.850$										$\lambda = 0.100$									
		5%	25%	50%	75%	95%	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%										
Estimators	Inst.																														
2S-GMM	4	0.5099	0.5729	0.6208	0.6602	0.7433	0.0517	0.074	0.1227	0.2343	0.3348	0.5290	0.6910	0.7718	0.8581	1.0000	0.0018	0.0522	0.0898	0.1275	0.1622	0.0018	0.0522	0.0898	0.1275	0.1622					
3S-EEL	4	0.5240	0.5854	0.6324	0.6788	0.7747	0.0510	0.0743	0.1111	0.2135	0.3235	0.4865	0.7246	0.818	0.9151	1.0000	0.0091	0.0664	0.1039	0.1283	0.1597	0.0091	0.0664	0.1039	0.1283	0.1597					
3SW-EEL	4	0.5195	0.5850	0.6317	0.6776	0.7706	0.0504	0.0741	0.1102	0.2125	0.3243	0.3621	0.7243	0.8159	0.9179	1.0000	0.0308	0.0676	0.1044	0.1281	0.1581	0.0308	0.0676	0.1044	0.1281	0.1581					
CUE	4	0.5235	0.5847	0.6293	0.6731	0.7744	0.0509	0.0725	0.1100	0.2160	0.3270	0.4616	0.7104	0.8042	0.9117	1.0000	0.0144	0.0580	0.1000	0.1305	0.1608	0.0144	0.0580	0.1000	0.1305	0.1608					
2S-GMM	8	0.4623	0.5234	0.5756	0.6224	0.6967	0.0535	0.0977	0.2009	0.2978	0.3635	0.5432	0.6368	0.6974	0.7658	0.8829	0.0089	0.038	0.0610	0.1065	0.1613	0.0089	0.038	0.0610	0.1065	0.1613					
3S-EEL	8	0.4971	0.5688	0.6155	0.6611	0.7626	0.0502	0.0761	0.1327	0.2452	0.3431	0.5502	0.6781	0.7629	0.8679	1.0000	0.0055	0.0477	0.0838	0.1233	0.1599	0.0055	0.0477	0.0838	0.1233	0.1599					
3SW-EEL	8	0.4958	0.5725	0.6187	0.6660	0.7662	0.0490	0.0735	0.1197	0.2362	0.3435	0.4338	0.6806	0.7702	0.8786	1.0000	0.0112	0.0496	0.0895	0.1237	0.1579	0.0112	0.0496	0.0895	0.1237	0.1579					
CUE	8	0.5065	0.5772	0.6214	0.6702	0.7910	0.0482	0.0733	0.1231	0.2340	0.3382	0.4099	0.6658	0.7662	0.8841	1.0000	0.0233	0.0439	0.0858	0.1263	0.1595	0.0233	0.0439	0.0858	0.1263	0.1595					
2S-GMM	12	0.4266	0.4819	0.5281	0.5802	0.648	0.0593	0.1628	0.2631	0.3379	0.3799	0.5245	0.6083	0.6546	0.7107	0.8061	0.0085	0.0327	0.0532	0.0934	0.1599	0.0085	0.0327	0.0532	0.0934	0.1599					
3S-EEL	12	0.4580	0.5302	0.5842	0.6365	0.7194	0.0507	0.0835	0.1837	0.2949	0.3643	0.5419	0.6391	0.7068	0.7887	0.9708	0.0089	0.0383	0.0632	0.1057	0.1566	0.0089	0.0383	0.0632	0.1057	0.1566					
3SW-EEL	12	0.4564	0.5385	0.5931	0.6452	0.7368	0.0483	0.0782	0.1574	0.2846	0.3632	0.5290	0.6419	0.7179	0.8093	1.0000	0.0060	0.0397	0.0679	0.1114	0.1552	0.0060	0.0397	0.0679	0.1114	0.1552					
CUE	12	0.4868	0.5692	0.6154	0.6710	0.8212	0.0452	0.0758	0.1424	0.2541	0.3506	0.2686	0.6280	0.7242	0.8504	1.0000	0.0489	0.0327	0.0697	0.1216	0.1619	0.0489	0.0327	0.0697	0.1216	0.1619					
2S-GMM	16	0.4009	0.4490	0.4871	0.539	0.6098	0.0876	0.2254	0.3175	0.3592	0.3910	0.5321	0.5925	0.6338	0.6760	0.7627	0.0140	0.0342	0.0565	0.0925	0.1582	0.0140	0.0342	0.0565	0.0925	0.1582					
3S-EEL	16	0.4178	0.4847	0.5439	0.6013	0.6922	0.0543	0.1244	0.2483	0.3325	0.3801	0.5412	0.6128	0.6661	0.7302	0.8786	0.0114	0.0340	0.0553	0.0886	0.1526	0.0114	0.0340	0.0553	0.0886	0.1526					
3SW-EEL	16	0.4172	0.4896	0.5551	0.6142	0.7071	0.0517	0.1039	0.2312	0.3276	0.3802	0.5301	0.6125	0.6714	0.7438	0.9201	0.0091	0.0339	0.0570	0.0917	0.1535	0.0091	0.0339	0.0570	0.0917	0.1535					
CUE	16	0.4161	0.5483	0.6055	0.6792	0.8765	0.0374	0.0807	0.1710	0.2858	0.3852	0.0012	0.6009	0.6946	0.8337	1.0000	0.1104	0.0254	0.0613	0.1156	0.1650	0.1104	0.0254	0.0613	0.1156	0.1650					
$T = 500$																															
2S-GMM	4	0.5683	0.6126	0.6388	0.665	0.7067	0.0617	0.0750	0.1053	0.1646	0.2478	0.6908	0.7681	0.8280	0.8849	0.9778	0.0458	0.0700	0.0975	0.1237	0.1448	0.0458	0.0700	0.0975	0.1237	0.1448					
3S-EEL	4	0.5718	0.6170	0.6444	0.6708	0.7130	0.0618	0.0747	0.0996	0.1585	0.2419	0.7075	0.7970	0.8498	0.9153	1.0000	0.0520	0.0834	0.1047	0.1222	0.1423	0.0520	0.0834	0.1047	0.1222	0.1423					
3SW-EEL	4	0.5720	0.6170	0.6443	0.6712	0.7149	0.0618	0.0747	0.0990	0.1577	0.2418	0.7037	0.8002	0.8511	0.9159	1.0000	0.0523	0.0858	0.1056	0.1218	0.1429	0.0523	0.0858	0.1056	0.1218	0.1429					
CUE	4	0.5720	0.6162	0.6421	0.6681	0.7092	0.0612	0.0744	0.1002	0.1587	0.2416	0.6953	0.7813	0.8444	0.9045	1.0000	0.0476	0.0736	0.1027	0.1258	0.1444	0.0476	0.0736	0.1027	0.1258	0.1444					
2S-GMM	8	0.5502	0.595	0.6259	0.6525	0.6925	0.0614	0.0792	0.1300	0.1902	0.2756	0.6709	0.7251	0.7816	0.8364	0.9254	0.0388	0.0571	0.0779	0.1079	0.1416	0.0388	0.0571	0.0779	0.1079	0.1416					
3S-EEL	8	0.5683	0.6145	0.6433	0.6700	0.7128	0.0613	0.0747	0.0998	0.1616	0.2554	0.6978	0.7776	0.8427	0.9101	1.0000	0.0480	0.0734	0.1014	0.1213	0.1425	0.0480	0.0734	0.1014	0.1213	0.1425					
3SW-EEL	8	0.5694	0.6147	0.6440	0.6709	0.7162	0.0615	0.0747	0.0991	0.1602	0.2553	0.6959	0.7852	0.8465	0.9143	1.0000	0.0493	0.0781	0.1038	0.1213	0.1420	0.0493	0.0781	0.1038	0.1213	0.1420					
CUE	8	0.5686	0.6141	0.6413	0.6670	0.7138	0.0608	0.0743	0.1011	0.1610	0.2550	0.6880	0.7642	0.8339	0.8991	1.0000	0.0445	0.0692	0.0986	0.1232	0.1430	0.0445	0.0692	0.0986	0.1232	0.1430					
2S-GMM	12	0.5340	0.5775	0.6108	0.6391	0.6799	0.0623	0.0907	0.1560	0.2127	0.2978	0.6470	0.6992	0.7421	0.7978	0.8767	0.0328	0.0488	0.0657	0.0927	0.1383	0.0328	0.0488	0.0657	0.0927	0.1383					
3S-EEL	12	0.5594	0.6074	0.6392	0.6655	0.7108	0.0608	0.0752	0.1067	0.1685	0.2678	0.6859	0.7563	0.8182	0.8880	1.0000	0.0434	0.0656	0.0902	0.1176	0.1411	0.0434	0.0656	0.0902	0.1176	0.1411					
3SW-EEL	12	0.5608	0.6090	0.6406	0.6674	0.7145	0.0607	0.0747	0.1019	0.1660	0.2652	0.6865	0.7628	0.8300	0.8999	1.0000	0.0438	0.0695	0.0974	0.1189	0.1421	0.0438	0.0695	0.0974	0.1189	0.1421					
CUE	12	0.5656	0.6122	0.6406	0.6671	0.7121	0.0605	0.0740	0.1032	0.1642	0.2634	0.6736	0.7483	0.8213	0.8954	1.0000	0.0405	0.0637	0.0942	0.1213	0.1430	0.0405	0.0637	0.0942	0.1213	0.1430					
2S-GMM	16	0.5162	0.5609	0.5954	0.6259	0.6669	0.0630	0.1110	0.1741	0.2377	0.3200	0.6341	0.6778	0.7189	0.7659	0.8415	0.0297	0.0444	0.0583	0.0822	0.1303	0.0297	0.0444	0.0583	0.0822	0.1303					
3S-EEL	16	0.5442	0.5982	0.6320	0.6604	0.7050	0.0608	0.0761	0.1164	0.1868	0.2885	0.6669	0.7294	0.7901	0.8617	0.9926	0.0387	0.0576	0.0784	0.1083	0.1409	0.0387	0.0576	0.0784	0.1083	0.1409					
3SW-EEL	16	0.5472	0.6020	0.6347	0.6636	0.7078	0.0600	0.0752	0.1110	0.1808	0.2857	0.6678	0.7382	0.8019	0.8788	1.0000	0.0391	0.0609	0.0846	0.1136	0.1414	0.0391	0.0609	0.0846	0.1136	0.1414					
CUE	16	0.5616	0.6106	0.6397	0.6673	0.7149	0.0604	0.0752	0.1069	0.1743	0.2737	0.6612	0.7366	0.8113	0.8954	1.0000	0.0368	0.0606	0.0905	0.1206	0.1432	0.0368	0.0606	0.0905	0.1206	0.1432					

**Table 5:** Monte Carlo simulations

$T = 160$		$\gamma_f = 0.650$		$\gamma_b = 0.300$		$\lambda = 0.100$		$\gamma_f = 0.850$		$\gamma_b = 0.100$		$\lambda = 0.100$	
Estimators	Inst.	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
2S-GMM	4	-0.0265	0.0763	-0.0208	0.1164	0.0120	0.0861	-0.0502	0.1054	0.0169	0.1210	-0.0033	0.0369
3S-EEL	4	-0.0160	0.0749	-0.0209	0.1133	0.0066	0.0813	-0.0166	0.1135	-0.0503	0.1186	0.0063	0.0357
3SW-EEL	4	-0.0159	0.0780	-0.0212	0.1138	0.0066	0.0825	-0.0158	0.1378	-0.0473	0.1416	0.0058	0.0388
CUE	4	-0.0196	0.0748	-0.0193	0.1151	0.0068	0.0828	-0.029	0.1071	-0.0323	0.1167	0.0034	0.0356
2S-GMM	8	-0.0664	0.1008	-0.0637	0.1350	0.0615	0.1103	-0.1055	0.1298	0.0974	0.1482	-0.0188	0.0415
3S-EEL	8	-0.0332	0.0851	-0.0347	0.1257	0.0224	0.0948	-0.044	0.1167	-0.0004	0.1286	-0.0007	0.0385
3SW-EEL	8	-0.0292	0.0863	-0.0291	0.1250	0.0179	0.0935	-0.0387	0.1352	-0.0161	0.1462	0.0009	0.0409
CUE	8	-0.0285	0.0846	-0.0331	0.1291	0.0186	0.6195	-0.0375	0.1242	-0.027	0.1346	0.0016	> 1
2S-GMM	12	-0.1058	0.1274	-0.0999	0.1529	0.0977	0.1323	-0.1414	0.1546	0.1427	0.1713	-0.0292	0.0443
3S-EEL	12	-0.0580	0.1059	-0.0613	0.1445	0.0561	0.1159	-0.0914	0.1338	0.0752	0.1503	-0.0155	0.0420
3SW-EEL	12	-0.0521	0.1066	-0.0549	0.1435	0.0458	0.1144	-0.0812	0.1405	0.0514	0.1562	-0.0108	0.0433
CUE	12	-0.0353	0.1069	-0.0560	0.1496	0.0352	> 1	-0.0547	0.1652	-0.0069	0.1693	-0.0030	> 1
2S-GMM	16	-0.1396	0.1551	-0.1307	0.1694	0.1286	0.1534	-0.1707	0.1776	0.1703	0.1869	-0.0333	0.0459
3S-EEL	16	-0.1002	0.1332	-0.1045	0.1642	0.0989	0.1402	-0.1314	0.1545	0.1310	0.1705	-0.0264	0.0448
3SW-EEL	16	-0.0899	0.1315	-0.0931	0.1631	0.0886	0.1381	-0.1273	0.1594	0.1222	0.1753	-0.0247	0.0457
CUE	16	-0.0454	0.1466	-0.0899	0.1731	0.0624	> 1	-0.0749	0.2065	0.0184	0.2026	-0.0069	> 1
$T = 500$													
2S-GMM	4	-0.0079	0.0429	-0.0072	0.0654	0.0034	0.0488	-0.0145	0.0659	0.0007	0.0857	-0.0007	0.0259
3S-EEL	4	-0.0039	0.0425	-0.0062	0.0635	-0.0003	0.0470	0.0030	0.0709	-0.0270	0.0806	0.0038	0.0243
3SW-EEL	4	-0.0038	0.0425	-0.0062	0.0634	-0.0001	0.0469	0.0036	0.0719	-0.0312	0.0811	0.0045	0.0243
CUE	4	-0.0048	0.0424	-0.0052	0.0642	0.0004	0.0475	-0.0061	0.0668	-0.0108	0.0839	0.0014	0.0254
2S-GMM	8	-0.0215	0.0494	-0.0189	0.0718	0.0177	0.0560	-0.0361	0.0701	0.0362	0.0933	-0.0094	0.0275
3S-EEL	8	-0.0050	0.0437	-0.0077	0.0650	0.0002	0.0483	0.0035	0.0719	-0.0273	0.0830	0.0037	0.0250
3SW-EEL	8	-0.0041	0.0436	-0.0074	0.0648	-0.0007	0.0480	0.0052	0.0731	-0.0326	0.0829	0.0044	0.0248
CUE	8	-0.0065	0.0439	-0.0089	0.0665	0.0020	0.0494	-0.0068	0.0688	-0.0129	0.0864	0.0015	0.0260
2S-GMM	12	-0.0342	0.0581	-0.0344	0.0795	0.0329	0.0644	-0.0594	0.0809	0.0668	0.1055	-0.0157	0.0295
3S-EEL	12	-0.0092	0.0468	-0.0117	0.0689	0.0046	0.0522	-0.0066	0.0721	-0.0099	0.0855	0.0012	0.0257
3SW-EEL	12	-0.0083	0.0463	-0.0109	0.0682	0.0025	0.0514	-0.0012	0.0736	-0.0232	0.0854	0.0032	0.0257
CUE	12	-0.0077	0.0454	-0.0106	0.0700	0.0033	0.0521	-0.0075	0.0716	-0.0112	0.0898	0.0009	0.0269
2S-GMM	16	-0.0462	0.0674	-0.0468	0.0886	0.0449	0.0736	-0.0788	0.0929	0.0910	0.1182	-0.0216	0.0315
3S-EEL	16	-0.0159	0.0521	-0.0185	0.0767	0.0140	0.0594	-0.0205	0.0747	0.0125	0.0912	-0.0045	0.0270
3SW-EEL	16	-0.0128	0.0512	-0.0152	0.0755	0.0093	0.0580	-0.0123	0.0761	-0.0053	0.0908	0.0002	0.0271
CUE	16	-0.0077	0.0479	-0.0154	0.0755	0.0078	0.0562	-0.0082	0.0753	-0.0153	0.0933	0.0019	0.0274

Note: For each parameter combination, the correlation parameter equals -0.5. For  $\gamma_f = 0.650$ ,  $\gamma_b = 0.300$  and  $\lambda = 0.100$ , the concentration parameter equals, respectively, 8.80 ( $T = 160$ ) and 27.27 ( $T = 500$ ). For  $\gamma_f = 0.850$ ,  $\gamma_b = 0.100$  and  $\lambda = 0.100$ , the concentration parameter equals, respectively, 29.17 ( $T = 160$ ) and 90.41 ( $T = 500$ ). The variance-covariance matrix of the moment conditions is estimated using the automatic lag selection procedure of Newey and West (1994). The number of simulations is 5,000.

**Table 6:** Monte Carlo simulations ( $\rho = -0.5$ ) —percentiles

$T = 160$		$\gamma_f = 0.650$										$\gamma_f = 0.850$										$\lambda = 0.100$									
		5%	25%	50%	75%	95%	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%	5%	25%	50%	75%	95%										
Estimators	Inst.																														
2S-GMM	4	0.4969	0.5762	0.6235	0.6637	0.7276	0.0574	0.0781	0.1120	0.1895	0.2922	0.6630	0.7424	0.7998	0.8695	1.0000	0.0422	0.0705	0.0967	0.1285	0.1590	0.6630	0.7424	0.7998	0.8695	1.0000	0.0422	0.0705	0.0967	0.1285	0.1590
3S-EEL	4	0.5077	0.5844	0.6340	0.6750	0.7447	0.0579	0.0781	0.1066	0.1794	0.2867	0.6792	0.7684	0.8334	0.9329	1.0000	0.0491	0.0816	0.1063	0.1298	0.1585	0.6792	0.7684	0.8334	0.9329	1.0000	0.0491	0.0816	0.1063	0.1298	0.1585
3SW-EEL	4	0.5072	0.5854	0.6341	0.6747	0.7452	0.0575	0.0780	0.1066	0.1796	0.2890	0.6644	0.7657	0.8342	0.9330	1.0000	0.0452	0.0814	0.1058	0.1295	0.1584	0.6644	0.7657	0.8342	0.9330	1.0000	0.0452	0.0814	0.1058	0.1295	0.1584
CUE	4	0.5056	0.5839	0.6304	0.6722	0.7426	0.0575	0.0777	0.1068	0.1819	0.2899	0.6736	0.7594	0.8210	0.9115	1.0000	0.0471	0.0770	0.1034	0.1293	0.1585	0.6736	0.7594	0.8210	0.9115	1.0000	0.0471	0.0770	0.1034	0.1293	0.1585
2S-GMM	8	0.4561	0.5304	0.5836	0.6328	0.6903	0.0592	0.0905	0.1615	0.2334	0.3266	0.6268	0.6926	0.7445	0.8027	0.9198	0.0339	0.0567	0.0812	0.1183	0.1598	0.6268	0.6926	0.7445	0.8027	0.9198	0.0339	0.0567	0.0812	0.1183	0.1598
3S-EEL	8	0.4817	0.5649	0.6168	0.6639	0.7317	0.0570	0.0803	0.1224	0.2042	0.3100	0.6483	0.7345	0.8060	0.8901	1.0000	0.0390	0.0706	0.0993	0.1281	0.1612	0.6483	0.7345	0.8060	0.8901	1.0000	0.0390	0.0706	0.0993	0.1281	0.1612
3SW-EEL	8	0.4832	0.5687	0.6208	0.6676	0.7402	0.0564	0.0788	0.1179	0.2008	0.3106	0.6371	0.7348	0.8113	0.8987	1.0000	0.0371	0.0716	0.1009	0.1280	0.1606	0.6371	0.7348	0.8113	0.8987	1.0000	0.0371	0.0716	0.1009	0.1280	0.1606
CUE	8	0.4905	0.5762	0.6215	0.6672	0.7444	0.0553	0.0789	0.1186	0.2010	0.3118	0.6415	0.7389	0.8125	0.9093	1.0000	0.0373	0.0701	0.1016	0.1302	0.1619	0.6415	0.7389	0.8125	0.9093	1.0000	0.0373	0.0701	0.1016	0.1302	0.1619
2S-GMM	12	0.4277	0.4938	0.5442	0.5979	0.6605	0.0632	0.1243	0.1977	0.2640	0.3475	0.6073	0.6610	0.7086	0.7584	0.8527	0.0294	0.0505	0.0708	0.1065	0.1584	0.6073	0.6610	0.7086	0.7584	0.8527	0.0294	0.0505	0.0708	0.1065	0.1584
3S-EEL	12	0.4472	0.5275	0.5920	0.6425	0.7133	0.0575	0.0858	0.1561	0.2432	0.3407	0.6198	0.6945	0.7586	0.8406	0.9973	0.0314	0.0572	0.0845	0.1198	0.1604	0.6198	0.6945	0.7586	0.8406	0.9973	0.0314	0.0572	0.0845	0.1198	0.1604
3SW-EEL	12	0.4453	0.5341	0.5979	0.6495	0.7225	0.0569	0.0837	0.1458	0.2377	0.3395	0.6113	0.6967	0.7688	0.8581	1.0000	0.0303	0.0591	0.0892	0.1243	0.1606	0.6113	0.6967	0.7688	0.8581	1.0000	0.0303	0.0591	0.0892	0.1243	0.1606
CUE	12	0.4620	0.559	0.6147	0.6679	0.7597	0.0538	0.0841	0.1352	0.2283	0.3338	0.6033	0.7069	0.7953	0.9067	1.0000	0.0271	0.0627	0.0970	0.1281	0.1637	0.6033	0.7069	0.7953	0.9067	1.0000	0.0271	0.0627	0.0970	0.1281	0.1637
2S-GMM	16	0.4010	0.4615	0.5104	0.5625	0.6313	0.0725	0.1634	0.2286	0.2932	0.3648	0.5914	0.6382	0.6793	0.7246	0.8092	0.0280	0.0467	0.0667	0.1016	0.1573	0.5914	0.6382	0.6793	0.7246	0.8092	0.0280	0.0467	0.0667	0.1016	0.1573
3S-EEL	16	0.4101	0.4842	0.5498	0.6117	0.6834	0.0604	0.1103	0.1989	0.2793	0.3638	0.5967	0.6597	0.7186	0.7888	0.9207	0.0284	0.0495	0.0736	0.1094	0.1597	0.5967	0.6597	0.7186	0.7888	0.9207	0.0284	0.0495	0.0736	0.1094	0.1597
3SW-EEL	16	0.4068	0.4902	0.5601	0.6206	0.6966	0.0590	0.1004	0.1886	0.2763	0.3655	0.5879	0.6593	0.7227	0.8031	0.9466	0.0261	0.0493	0.0753	0.1122	0.1593	0.5879	0.6593	0.7227	0.8031	0.9466	0.0261	0.0493	0.0753	0.1122	0.1593
CUE	16	0.4046	0.5388	0.6046	0.6658	0.8045	0.0547	0.0884	0.1624	0.2616	0.3948	0.5483	0.6822	0.7751	0.8901	1.0000	0.0167	0.0560	0.0931	0.1281	0.1759	0.5483	0.6822	0.7751	0.8901	1.0000	0.0167	0.0560	0.0931	0.1281	0.1759
$T = 500$																															
2S-GMM	4	0.5648	0.6123	0.6421	0.6669	0.7000	0.0661	0.0777	0.1034	0.1449	0.2060	0.7445	0.7937	0.8355	0.8821	0.9585	0.0615	0.0796	0.0993	0.1216	0.1437	0.7445	0.7937	0.8355	0.8821	0.9585	0.0615	0.0796	0.0993	0.1216	0.1437
3S-EEL	4	0.5678	0.6164	0.6461	0.6711	0.7058	0.0662	0.0780	0.0997	0.1408	0.2049	0.7562	0.8095	0.8530	0.9053	1.0000	0.0656	0.0861	0.1038	0.1233	0.1440	0.7562	0.8095	0.8530	0.9053	1.0000	0.0656	0.0861	0.1038	0.1233	0.1440
3SW-EEL	4	0.5680	0.6166	0.6462	0.6710	0.7058	0.0662	0.0780	0.0999	0.1410	0.2042	0.7564	0.8095	0.8536	0.9067	1.0000	0.0658	0.0864	0.1045	0.1234	0.1443	0.7564	0.8095	0.8536	0.9067	1.0000	0.0658	0.0864	0.1045	0.1234	0.1443
CUE	4	0.5669	0.6157	0.6452	0.6695	0.7037	0.0662	0.0777	0.1004	0.1425	0.2036	0.7481	0.8027	0.8439	0.8904	0.9757	0.0637	0.0817	0.1014	0.1226	0.1437	0.7481	0.8027	0.8439	0.8904	0.9757	0.0637	0.0817	0.1014	0.1226	0.1437
2S-GMM	8	0.5514	0.5965	0.6285	0.6564	0.6924	0.0663	0.0815	0.1177	0.1609	0.2181	0.7240	0.7715	0.8139	0.8547	0.9255	0.0560	0.0734	0.0906	0.1155	0.1425	0.7240	0.7715	0.8139	0.8547	0.9255	0.0560	0.0734	0.0906	0.1155	0.1425
3S-EEL	8	0.5656	0.6126	0.6450	0.6716	0.7068	0.0661	0.0780	0.1002	0.1444	0.2073	0.7478	0.8058	0.8535	0.9039	1.0000	0.0645	0.0843	0.1037	0.1231	0.1446	0.7478	0.8058	0.8535	0.9039	1.0000	0.0645	0.0843	0.1037	0.1231	0.1446
3SW-EEL	8	0.5660	0.6138	0.6459	0.6720	0.7073	0.0661	0.0779	0.0993	0.1438	0.2069	0.7497	0.8076	0.8552	0.9064	1.0000	0.0652	0.0858	0.1044	0.1237	0.1448	0.7497	0.8076	0.8552	0.9064	1.0000	0.0652	0.0858	0.1044	0.1237	0.1448
CUE	8	0.5661	0.6125	0.6435	0.6697	0.7041	0.0659	0.0779	0.1020	0.1463	0.2072	0.7420	0.7980	0.8432	0.8915	0.9741	0.0618	0.0810	0.1015	0.1227	0.1444	0.7420	0.7980	0.8432	0.8915	0.9741	0.0618	0.0810	0.1015	0.1227	0.1444
2S-GMM	12	0.5369	0.5816	0.6158	0.6452	0.6840	0.0668	0.0884	0.1329	0.1726	0.2337	0.7046	0.7520	0.7906	0.8314	0.8988	0.0519	0.0684	0.0843	0.1092	0.1415	0.7046	0.7520	0.7906	0.8314	0.8988	0.0519	0.0684	0.0843	0.1092	0.1415
3S-EEL	12	0.5562	0.6079	0.6408	0.6678	0.7063	0.0662	0.0786	0.1046	0.1512	0.2144	0.7395	0.7966	0.8434	0.8924	0.9856	0.0618	0.0814	0.1012	0.1224	0.1442	0.7395	0.7966	0.8434	0.8924	0.9856	0.0618	0.0814	0.1012	0.1224	0.1442
3SW-EEL	12	0.5570	0.6096	0.6417	0.6685	0.7069	0.0660	0.0784	0.1025	0.1495	0.2131	0.7420	0.8003	0.8488	0.8996	1.0000	0.0628	0.0834	0.1032	0.1234	0.1450	0.7420	0.8003	0.8488	0.8996	1.0000	0.0628	0.0834	0.1032	0.1234	0.1450
CUE	12	0.5592	0.6098	0.6423	0.6693	0.7048	0.0655	0.0776	0.1033	0.1500	0.2137	0.7341	0.7947	0.8425	0.8919	0.9778	0.0596	0.0800	0.1009	0.1240	0.1448	0.7341	0.7947	0.8425	0.8919	0.9778	0.0596	0.0800	0.1009	0.1240	0.1448
2S-GMM	16	0.5232	0.5677	0.6038	0.6330	0.6741	0.0677	0.1021	0.1449	0.1881	0.2454	0.6908	0.7365	0.7712	0.8124	0.8713	0.0484	0.0641	0.0784	0.1025	0.1394	0.6908	0.7365	0.7712	0.8124	0.8713	0.0484	0.0641	0.0784	0.1025	0.1394
3S-EEL	16	0.5474	0.5970	0.6341	0.6634	0.7017	0.0663	0.0800	0.1140	0.1618	0.2255	0.7240	0.7818	0.8295	0.8833	0.9711	0.0580	0.0766	0.0955	0.1198	0.1433	0.7240	0.7818	0.8295	0.8833	0.9711	0.0580	0.0766	0.0955	0.1198	0.1433
3SW-EEL	16	0.5488	0.5993	0.6372	0.6656	0.7040	0.0663	0.0795	0.1093	0.1594	0.2233	0.7266	0.7877	0.8377	0.8926	0.9910	0.0593	0.0790	0.1002	0.1223	0.1456	0.7266	0.7877	0.8377	0.8926	0.9910	0.0593	0.0790	0.1002	0.1223	0.1456
CUE	16	0.5562	0.6066	0.6423	0.6684	0.7068	0.0659	0.0790	0.1078	0.1541	0.2220	0.7262	0.7911	0.8418	0.8944	0.9849	0.0571	0.0795	0.1019	0.1236	0.1443	0.7262	0.7911	0.8418	0.8944	0.9849	0.0571	0.0795	0.1019	0.1236	0.1443

**Table 7:** Monte Carlo simulations

$T = 160$		$\gamma_f = 0.550$		$\gamma_b = 0.400$		$\lambda = 0.100$		$\gamma_f = 0.750$		$\gamma_b = 0.200$		$\lambda = 0.100$	
Estimators	Inst.	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
$\rho = 0$													
2S-GMM	4	-0.0724	0.1454	-0.0250	0.0542	0.0634	0.1268	-0.0261	0.0854	-0.0105	0.1281	0.0064	0.0559
3S-EEL	4	-0.0218	0.1259	-0.0147	0.0496	0.0213	0.1090	-0.0034	0.1002	-0.0467	0.1323	0.0132	0.0547
3SW-EEL	4	-0.0408	0.1796	-0.0203	0.0610	0.0373	0.1530	-0.0043	0.1105	-0.0436	0.1401	0.0129	0.0559
CUE	4	-0.0168	0.1261	-0.0144	0.0512	0.0137	0.1081	-0.0142	0.0976	-0.0315	0.1349	0.0102	0.0560
2S-GMM	8	-0.1956	0.2345	-0.0636	0.0792	0.1739	0.2055	-0.0632	0.0891	0.0214	0.1256	-0.0016	0.0577
3S-EEL	8	-0.1067	0.1840	-0.0351	0.0649	0.0928	0.1604	-0.0234	0.0978	-0.0179	0.1324	0.0070	0.0562
3SW-EEL	8	-0.0948	0.1946	-0.0319	0.0662	0.0805	0.1675	-0.0192	0.1088	-0.0252	0.1407	0.0089	0.0576
CUE	8	-0.0411	0.1644	-0.0208	0.0668	0.0366	> 1	-0.0197	0.1113	-0.0294	0.1459	0.011	> 1
2S-GMM	12	-0.2642	0.2913	-0.0845	0.0970	0.2314	0.2559	-0.0911	0.1028	0.0423	0.1310	0.0003	0.0616
3S-EEL	12	-0.1901	0.2484	-0.0603	0.0845	0.1661	0.2178	-0.0537	0.0979	0.0158	0.1358	-0.0016	0.0601
3SW-EEL	12	-0.1607	0.2346	-0.0511	0.0801	0.1383	0.2053	-0.0494	0.1031	0.0106	0.1397	0.0002	0.0601
CUE	12	-0.0812	0.2167	-0.0321	0.0956	0.0704	> 1	-0.0283	0.1294	-0.0299	0.1585	0.0117	> 1
$\rho = 0.5$													
2S-GMM	4	-0.0784	0.1533	-0.0305	0.0352	0.0752	0.1393	-0.0782	0.1715	0.0447	0.1894	-0.0102	0.0534
3S-EEL	4	-0.0302	0.1367	-0.0187	0.0336	0.0312	0.1222	-0.0320	0.1915	-0.0142	0.2025	0.0039	0.0554
3SW-EEL	4	-0.0524	0.1866	-0.0248	0.0369	0.0500	0.1669	-0.0341	0.2174	-0.0147	0.2252	0.0044	0.0599
CUE	4	-0.0214	0.1319	-0.0173	0.0338	0.0252	0.1289	-0.0458	0.2056	-0.0209	0.2198	0.0001	0.0599
2S-GMM	8	-0.2060	0.2489	-0.0744	0.0322	0.1937	0.2299	-0.1526	0.1834	0.1657	0.2035	-0.039	0.0552
3S-EEL	8	-0.1212	0.2019	-0.0441	0.0384	0.1133	0.1829	-0.0871	0.1826	0.0713	0.1991	-0.0162	0.0552
3SW-EEL	8	-0.1039	0.1998	-0.0403	0.0393	0.0999	0.1805	-0.0798	0.2087	0.0463	0.2222	-0.0105	0.0595
CUE	8	-0.0626	0.1811	-0.0260	0.0392	0.0562	> 1	-0.0838	0.2229	0.0648	0.2401	-0.0142	0.0812
2S-GMM	12	-0.2766	0.3047	-0.0998	0.0293	0.2570	0.2841	-0.1954	0.2081	0.2013	0.2212	-0.0468	0.0573
3S-EEL	12	-0.2040	0.2619	-0.0735	0.0370	0.1908	0.2411	-0.1432	0.1836	0.1550	0.2036	-0.0368	0.0554
3SW-EEL	12	-0.1760	0.2420	-0.0624	0.0409	0.1639	0.2227	-0.1321	0.1926	0.1377	0.2098	-0.0321	0.0565
CUE	12	-0.0926	0.2304	-0.0386	0.0482	0.0871	> 1	-0.1258	0.2527	0.1283	0.2695	-0.0303	> 1
$\rho = -0.5$													
2S-GMM	4	-0.0924	0.1721	-0.0303	0.0594	0.0764	0.1454	-0.027	0.0788	-0.0090	0.1200	0.005	0.0527
3S-EEL	4	-0.0326	0.148	-0.0172	0.0537	0.0267	0.1250	-0.0075	0.0874	-0.0345	0.1221	0.0100	0.0517
3SW-EEL	4	-0.0547	0.1943	-0.0235	0.0644	0.0489	0.1625	-0.0078	0.0912	-0.0355	0.1254	0.0107	0.0522
CUE	4	-0.0242	0.1449	-0.0162	0.0555	0.0223	0.1568	-0.0155	0.0851	-0.0230	0.1233	0.0078	0.0523
2S-GMM	8	-0.2312	0.2697	-0.0702	0.0852	0.1954	0.2271	-0.0575	0.0824	0.0116	0.1196	0.0035	0.0559
3S-EEL	8	-0.1380	0.2190	-0.0426	0.0723	0.1159	0.1847	-0.0231	0.0862	-0.0221	0.1245	0.0079	0.0547
3SW-EEL	8	-0.1213	0.2273	-0.0383	0.0743	0.1019	0.1910	-0.0199	0.0914	-0.0281	0.1291	0.0093	0.0553
CUE	8	-0.0583	0.1921	-0.0246	0.0741	0.0501	> 1	-0.0175	0.0924	-0.0343	0.1308	0.0106	> 1
2S-GMM	12	-0.3020	0.3262	-0.0918	0.1010	0.2530	0.2741	-0.0815	0.0949	0.0298	0.1222	0.0027	0.0575
3S-EEL	12	-0.2361	0.2837	-0.0719	0.0903	0.1999	0.2391	-0.0472	0.0906	0.0020	0.1274	0.0042	0.0573
3SW-EEL	12	-0.2058	0.2726	-0.0638	0.0871	0.1768	0.2295	-0.0406	0.0934	-0.0058	0.1311	0.0057	0.0581
CUE	12	-0.1039	0.2411	-0.0364	0.1033	0.0863	> 1	-0.0240	0.1177	-0.0444	0.1484	0.0147	> 1

Note: For each parameter combination, the sample size is 160. For  $\gamma_f = 0.550$ ,  $\gamma_b = 0.400$  and  $\lambda = 0.100$ , the concentration parameter equals, respectively, 2.28 ( $\rho = 0$ ), 1.95 ( $\rho = 0.5$ ) and 1.97 ( $\rho = -0.5$ ). For  $\gamma_f = 0.750$ ,  $\gamma_b = 0.200$  and  $\lambda = 0.100$ , the concentration parameter equals, respectively, 18.07 ( $\rho = 0$ ), 14.40 ( $\rho = 0.5$ ) and 22.06 ( $\rho = -0.5$ ). The variance-covariance matrix of the moment conditions is estimated using the automatic lag selection procedure of Newey and West (1994). The number of simulations is 5,000.